

Electron-matter interactions: Plasmonics & nano-optics

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“Depending on the situation, surface plasmons must be regarded as many-body electron states or as electromagnetic waves, thus requiring either a quantum or a classical treatment with the possible inclusion of relativistic effects. This variety is a major obstacle to the establishment of a general theoretical treatment.

Even more crucially, surface plasmons suffer from huge ohmic losses in the metal which lead to very short lifetimes (typically a few femtoseconds). Theoretically, this presence of dissipation dramatically hardens the theoretical description of surface plasmons.”

Hugo Lourenço-Martins, PhD thesis

EPFL Contents

- Kramers-Kronig analysis and band gap measurements in visible light regime
- An evanescent probe
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- Coupling of plasmonic nanoparticles: EELS vs planewave illumination;
imaging of hybridisation and bonding modes
- Plasmon damping; scattering and absorption
- Dielectrics and bound-states-in-the-continuum

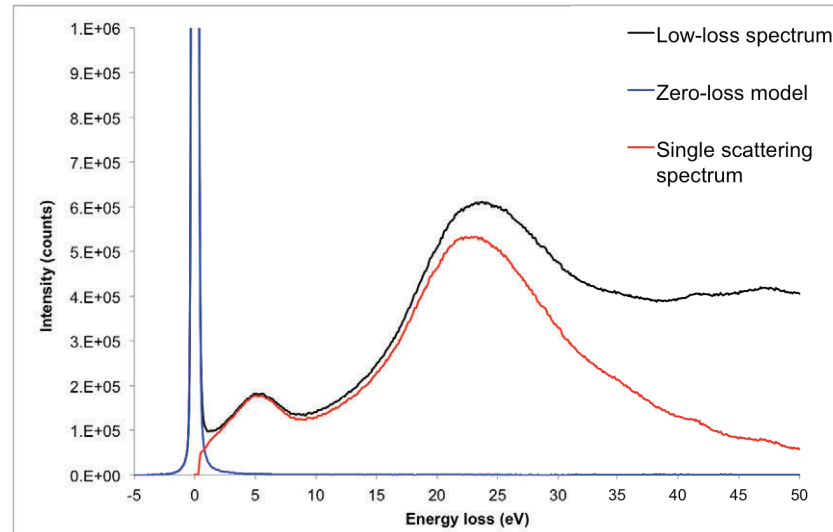
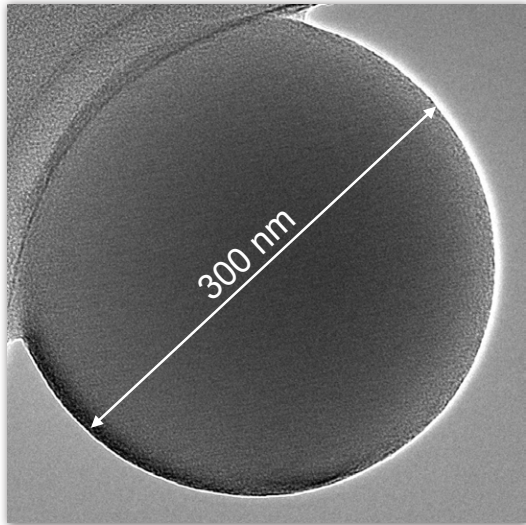
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EPFL EELS $\iff$ Optical response

- Deep relationship between EELS and light-optical response
- Example: determination of optical properties of atmospheric amorphous carbon spheres using Kramers-Kronig analysis of SSD EEL spectrum via:

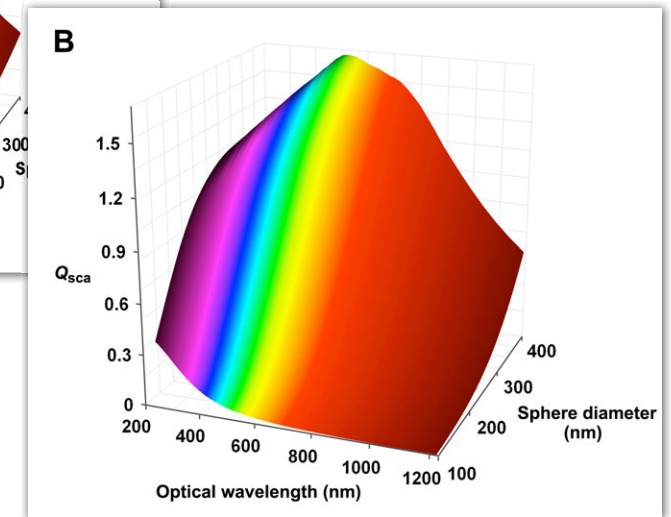
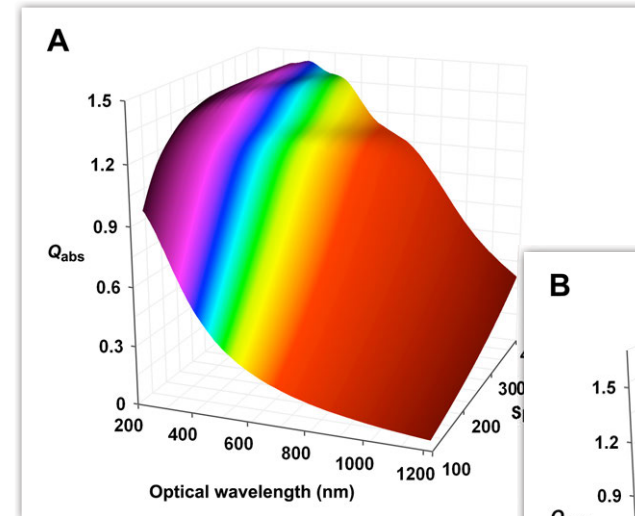
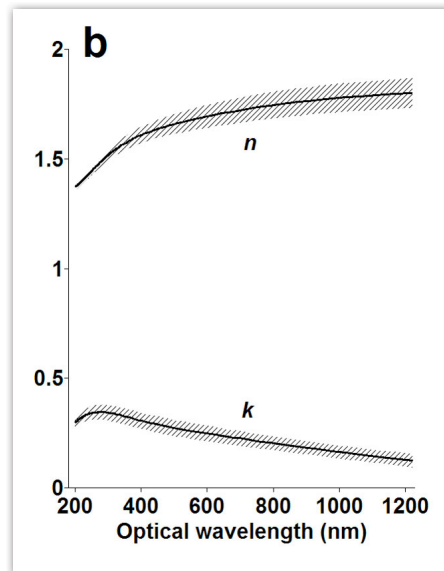
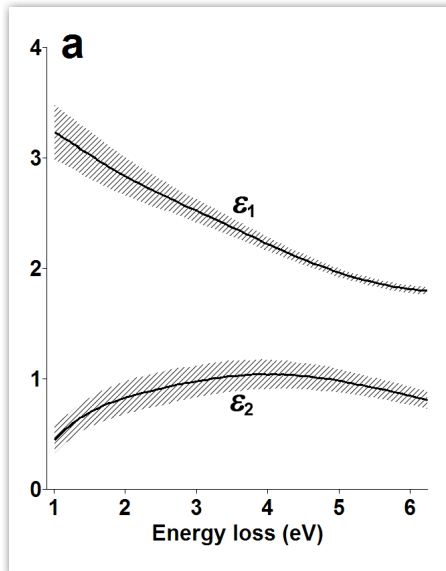
$$\Gamma_{\text{bulk}}^{\text{NR}}(\omega) = \frac{2e^2 L}{\pi \hbar v^2} \text{Im} \left\{ -\frac{1}{\epsilon} \right\} \ln \left(\frac{q_c v}{\omega} \right)$$



Alexander et al. *Science* **321** (2008) 833–836

EPFL EELS $\iff$ Optical response

- Kramers-Kronig normalisation using thickness of spheres



Property	Carbon sphere	Soot carbon
$N = n - ik$	$1.27 - 0.27i$	$1.95 - 0.79i$
Average spherule diameter (nm)	230	32
Number of spherules	1	55
$\sigma_{abs} (\times 10^{-14} \text{ m}^2)$	4.0	0.8
$\sigma_{sca} (\times 10^{-14} \text{ m}^2)$	3.2	0.2
ω_0	0.44	0.23
$\rho (\text{g cm}^{-3})$	$\sim 1.4\text{--}1.6$	$1.8\text{--}2.0$
MAE ($\text{m}^2 \text{ g}^{-1}$)	$\sim 3.6\text{--}4.1$	$4.3\text{--}4.8$
MSE ($\text{m}^2 \text{ g}^{-1}$)	$\sim 3.2\text{--}3.7$	$1.3\text{--}1.4$
$\hat{A}_{abs}$	~ 1.5	~ 1

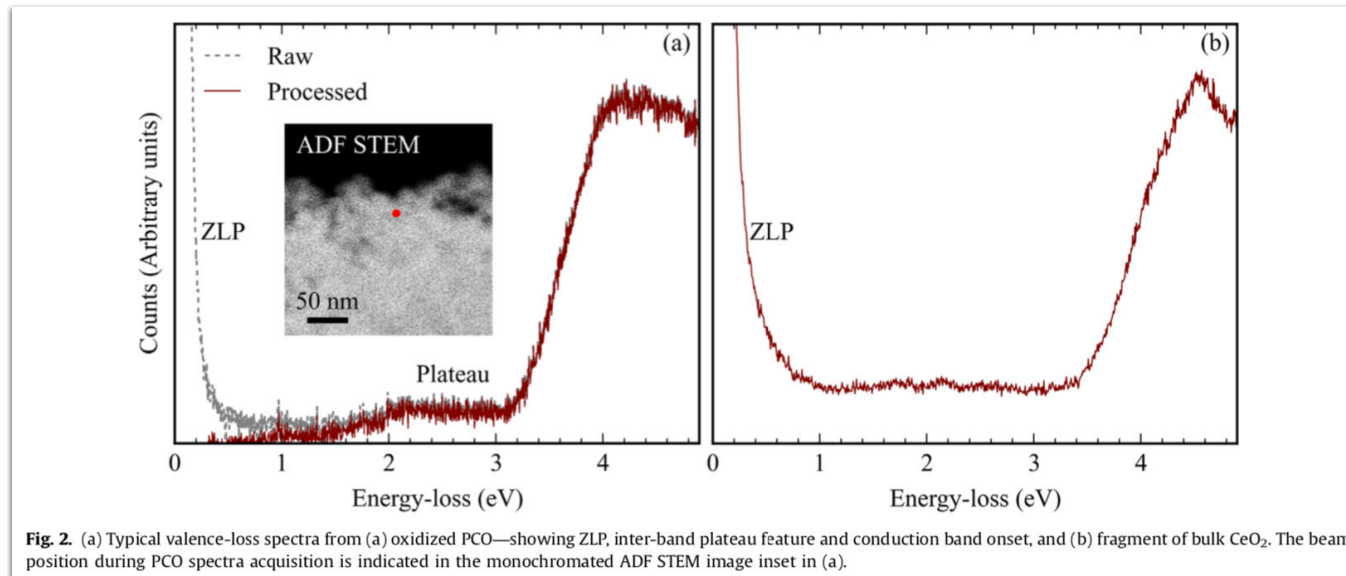
Alexander et al. *Science* **321** (2008) 833–836

Mie theory: Bohren and Huffman

“Absorption and scattering of light by small particles”

EPFL Band gap measurements

- In band gap $\epsilon'' \rightarrow 0$ therefore we should have $\Gamma_{\text{bulk}} \rightarrow 0$
- Measurements fraught with difficulties (Cherenkov radiation, surface damage/oxidation, surface plasmons, removal of ZLP...). See papers by M. Stöger-Pollach
- Example of good study: Bowman et al. *Ultramicroscopy* **167** (2016) 5–10:



- What is the spatial resolution of such a measurement?

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EPFL An evanescent probe

- Point charge moving in vacuum: evanescent source of radiation
⇒ can explore regions of momentum-space that lie outside the light cone
- Electric field of swift charged particle moving in homogeneous medium (distance r , frequency ω , time t):

$$\vec{E}(\vec{r}, t) = \int \frac{d\omega}{2\pi} \vec{E}(\vec{r}, \omega) e^{-i\omega t}$$

- In spectral domain, direct solution of Maxwell's equations yields:

$$\vec{E}(\vec{r}, \omega) = \frac{ie}{\pi} \int d\vec{q} \frac{\vec{q}/\epsilon - k\vec{v}/c}{q^2 - k^2\epsilon} e^{i\vec{q}\cdot\vec{r}} \delta(\omega - \vec{q}\cdot\vec{v})$$

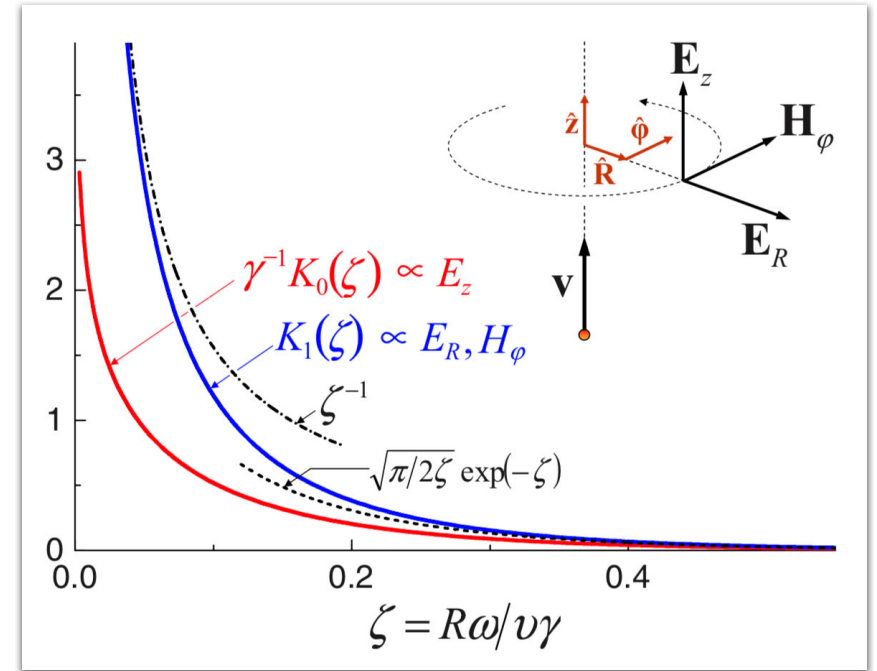
for wave vector $\vec{q}$ where $k = \omega/c$ is the light wave number in free space and ϵ is the dielectric function of the homogeneous medium

F.J. García de Abajo “Optical excitations in electron microscopy,” Rev. Mod. Phys. **82** (2010) 209–275

EPFL An evanescent probe

- Applying cartesian coordinates & with $v < c$ integral yields:

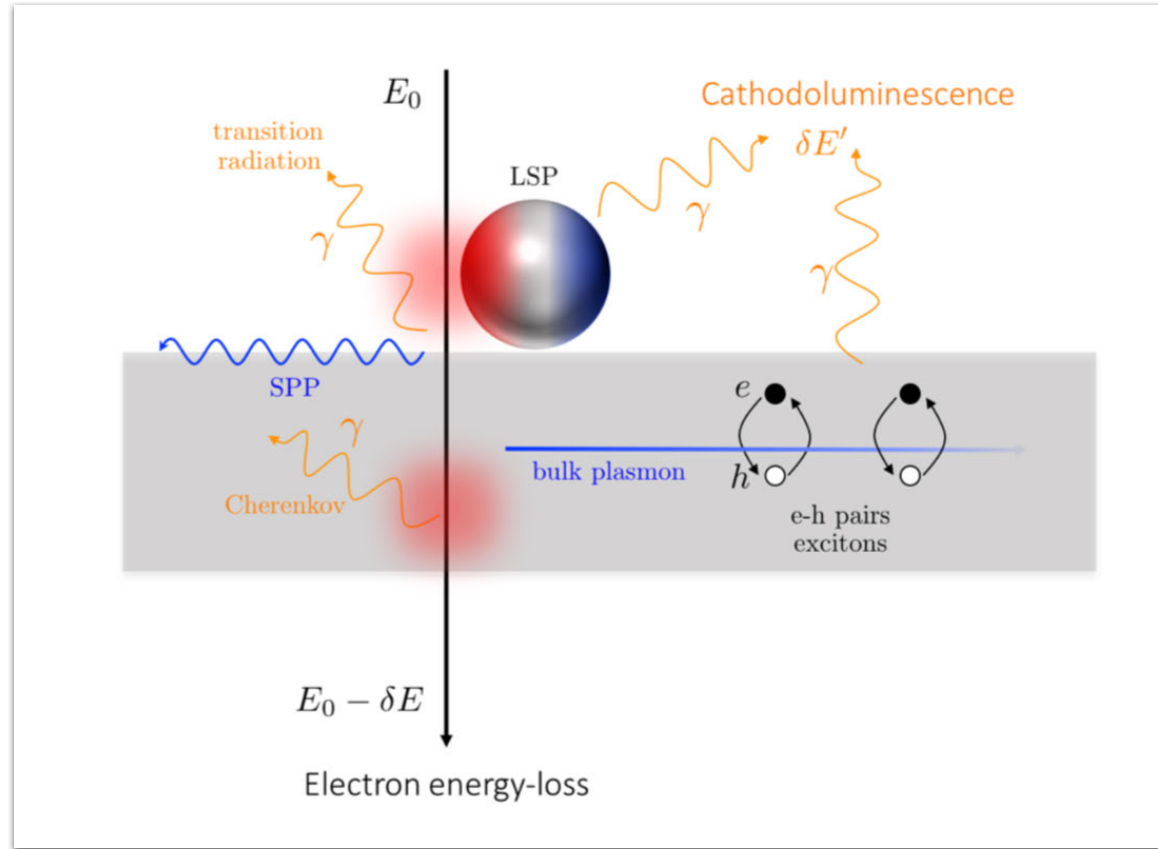
$$\bar{\mathbf{E}}(r, \omega) = \frac{2e\omega}{v^2\gamma\epsilon} e^{i\omega z/v} \left[\frac{i}{\gamma} K_0\left(\frac{\omega R}{v\gamma}\right) \hat{\mathbf{z}} - K_1\left(\frac{\omega R}{v\gamma}\right) \hat{\mathbf{R}}_{\perp} \right]$$



- Non vanishing components decay exponentially $\Rightarrow$ response delocalisation
Greater $v \Rightarrow$ field extends further

F.J. García de Abajo "Optical excitations in electron microscopy," Rev. Mod. Phys. **82** (2010) 209–275

EPFL Probe-sample interactions



- e-beam does not need to penetrate sample to stimulate surface excitation
⇒ measurements in “aloof” geometry

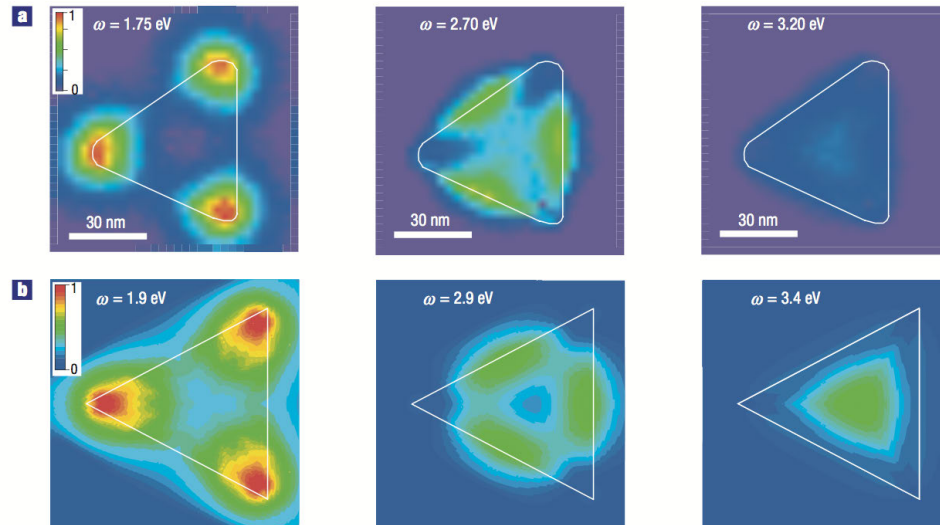
Figure from: Hugo Lourenço-Martins, PhD thesis

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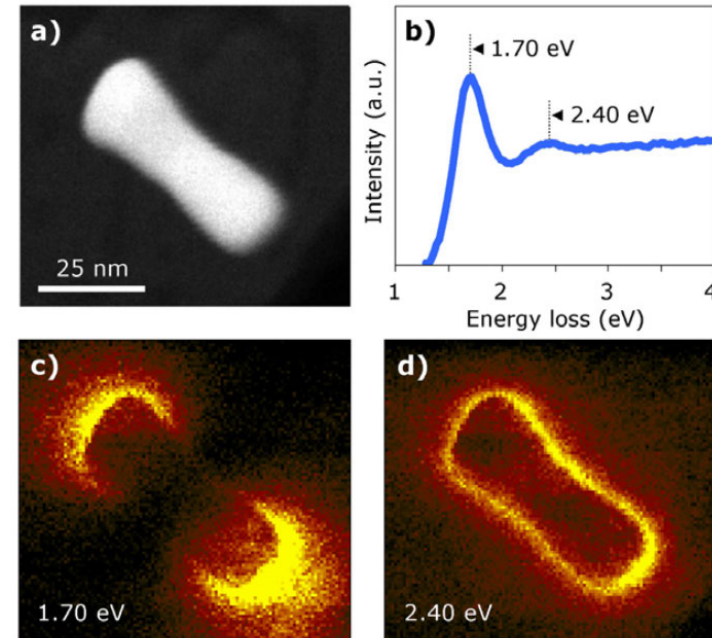
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EPFL First EELS mapping of LSPRs

- EELS measurement of localised surface plasmon resonance (LSPR) eigenmodes



Nelayah et al. “Mapping surface plasmons on a single metallic nanoparticle,” *Nature Phys.* **3** (2007) 348–353



Bosman et al. “Mapping surface plasmons at the nanometre scale with an electron beam,” *Nanotechnology* **18** (2007) 165505

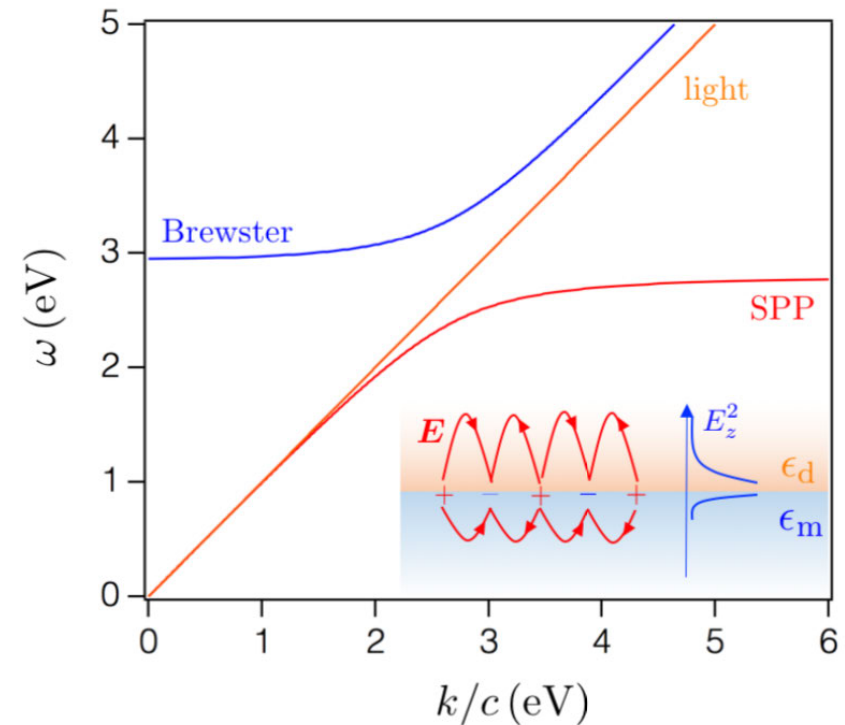
- Looks impressive: spatial resolution $\sim \lambda/(30-40)$ – *but what are we actually measuring?*

EPFL Surface plasmons(-polaritons)

- SPs: collective charge density waves of e-s propagating at interface associated to strongly confined EM fields
- Consider interface of 2 semi-infinite media with dielectric constants ϵ_m and ϵ_d
- Solution of Helmholtz wave eq. and continuity relations are surface plasmons (SPs):

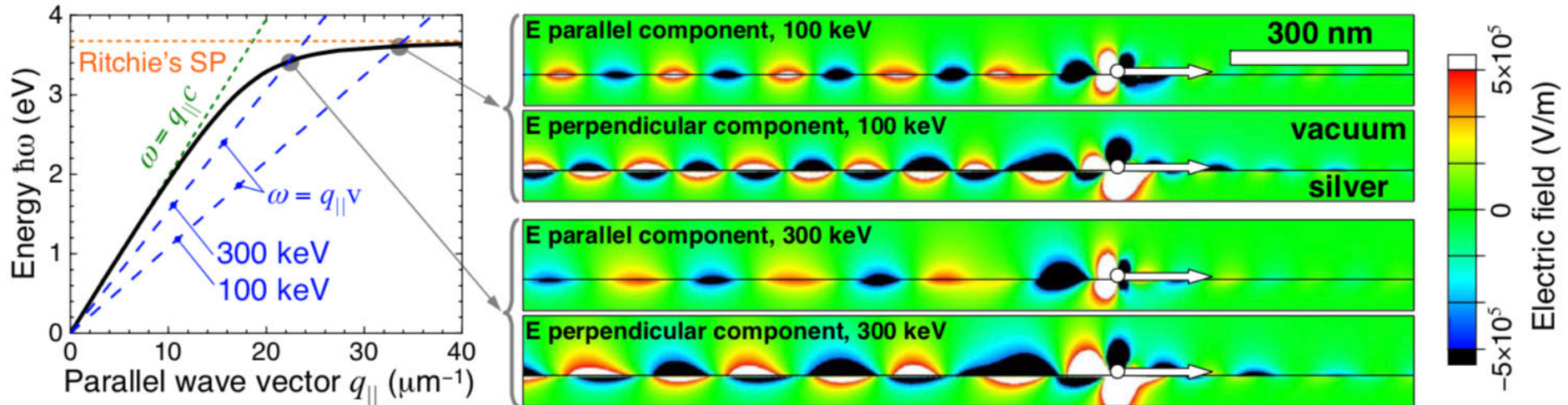
$$\vec{E}_j = \begin{pmatrix} E_{x,j} \\ 0 \\ E_{z,j} \end{pmatrix} e^{i(k_x x - \omega t)} e^{ik_{z,j} z}$$

- Dispersion relation: $k_{SP} = \frac{\omega}{c} \sqrt{\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d}}$



EPFL Surface plasmons(-polaritons)

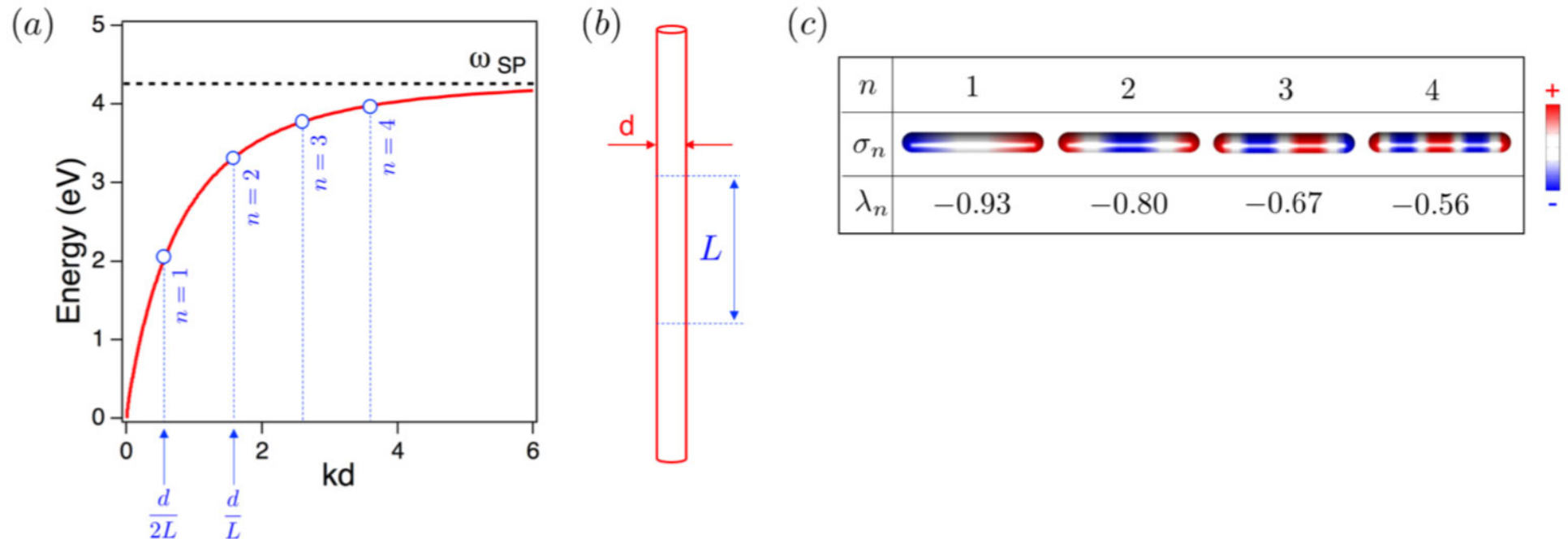
- Semi-infinite interface: SPs are traveling waves with evanescent field in z direction: surface plasmon-polaritons (SPPs)
- SPPs: TM modes; cannot couple directly with light
- Modelling of excitation by fast e-s moving parallel to Ag/vacuum interface at impact parameter of 10 nm:



F.J. García de Abajo "Optical excitations in electron microscopy," Rev. Mod. Phys. **82** (2010) 209–275

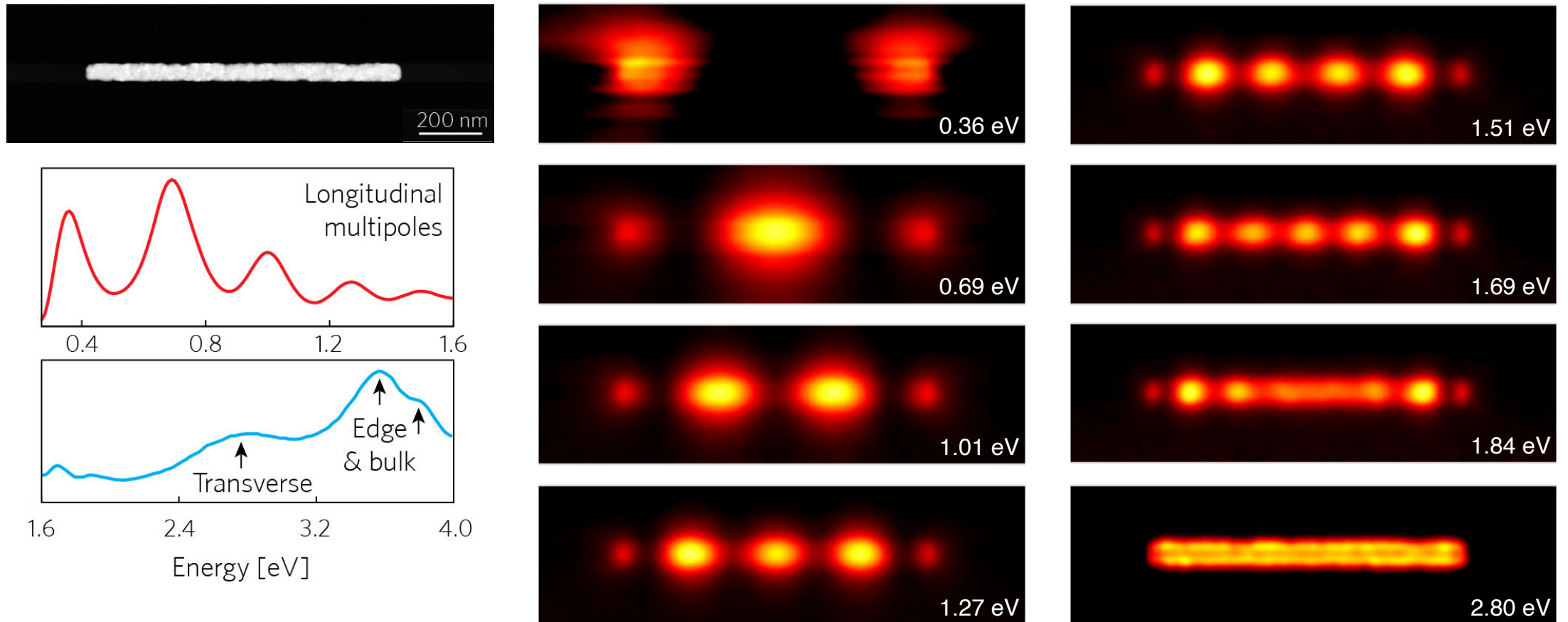
EPFL Localised surface plasmon resonances

- Confine SP on nanoparticle: set up a stationary standing wave
- Example: modelling of 400 x 30 nm Ag rod



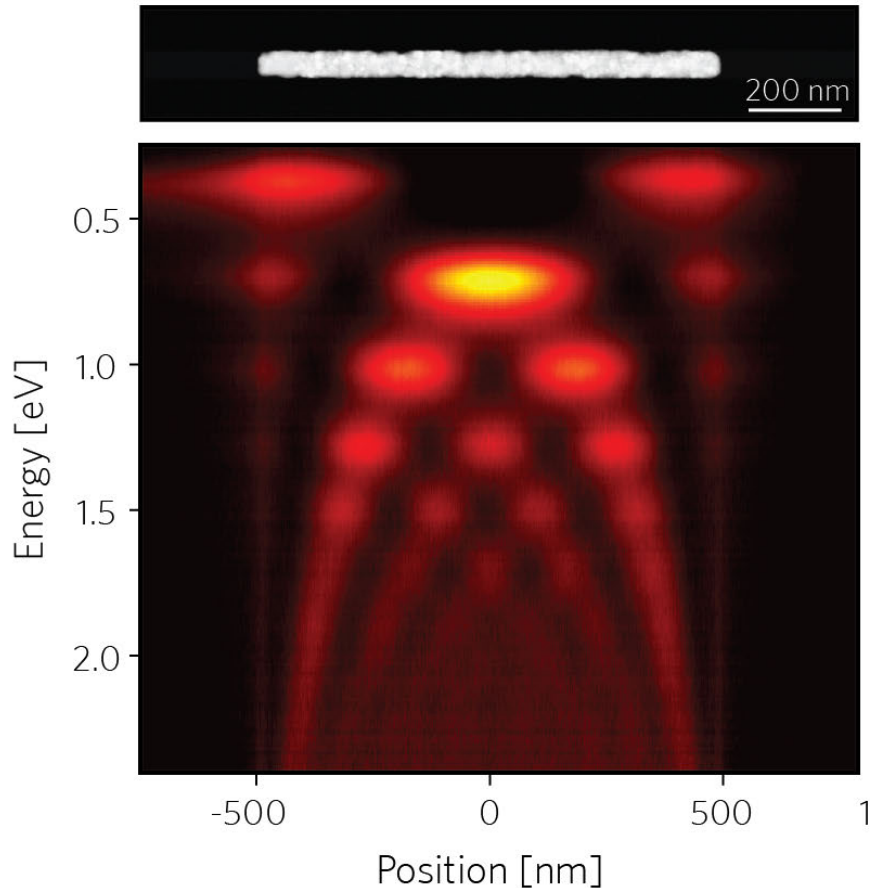
EPFL LSPRs on nanorod

- EELS measurements of 1000 x 50 nm Ag rod
- Strong EELS signal when $\vec{E}$ of probe excites LSPR with energy $\Delta E = hc/\lambda$



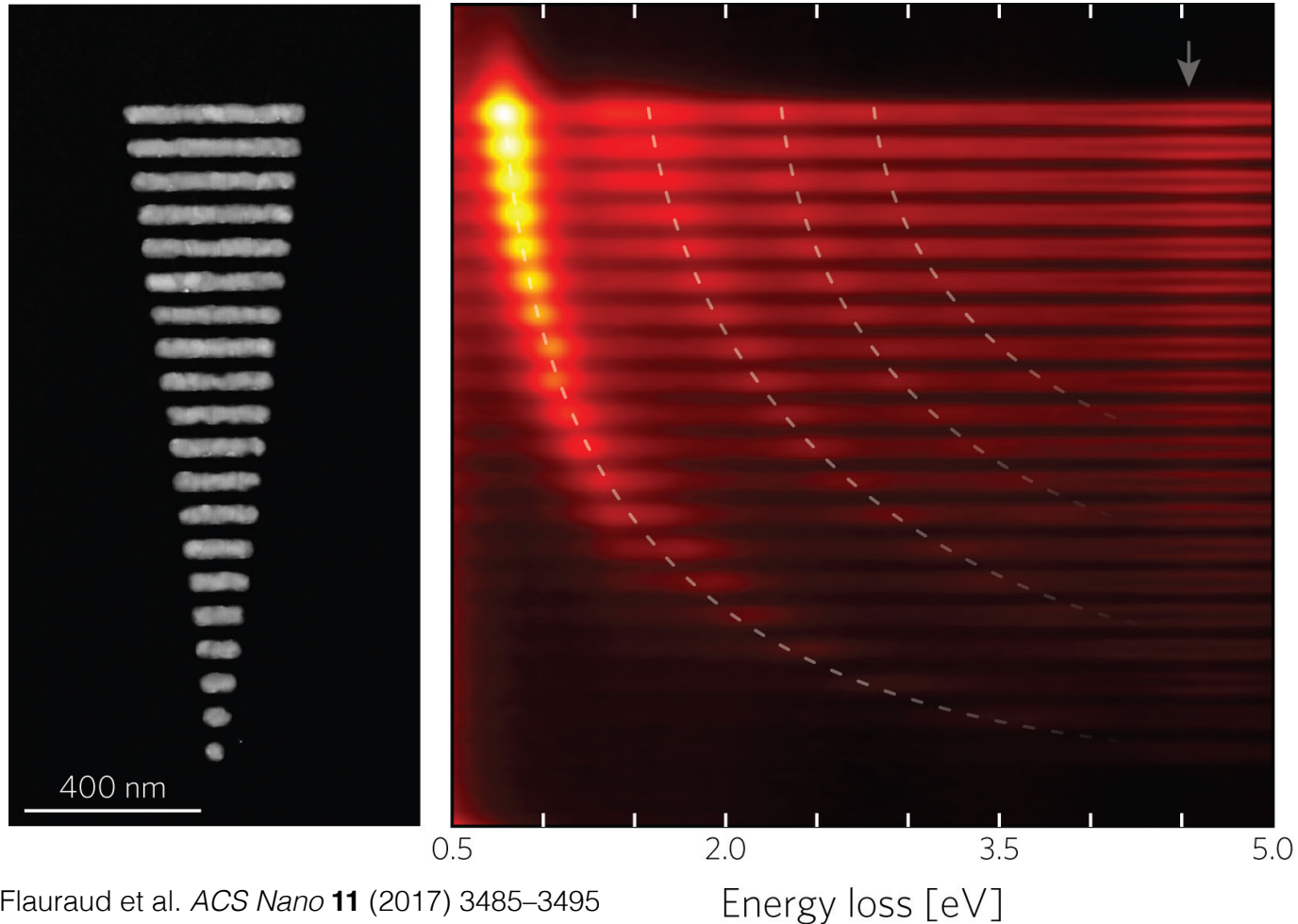
EPFL LSPRs on nanorod

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EPFL LSPRs on Al nanorods – dispersion curve

- Red-shift of resonance energies with increasing length of nanorod



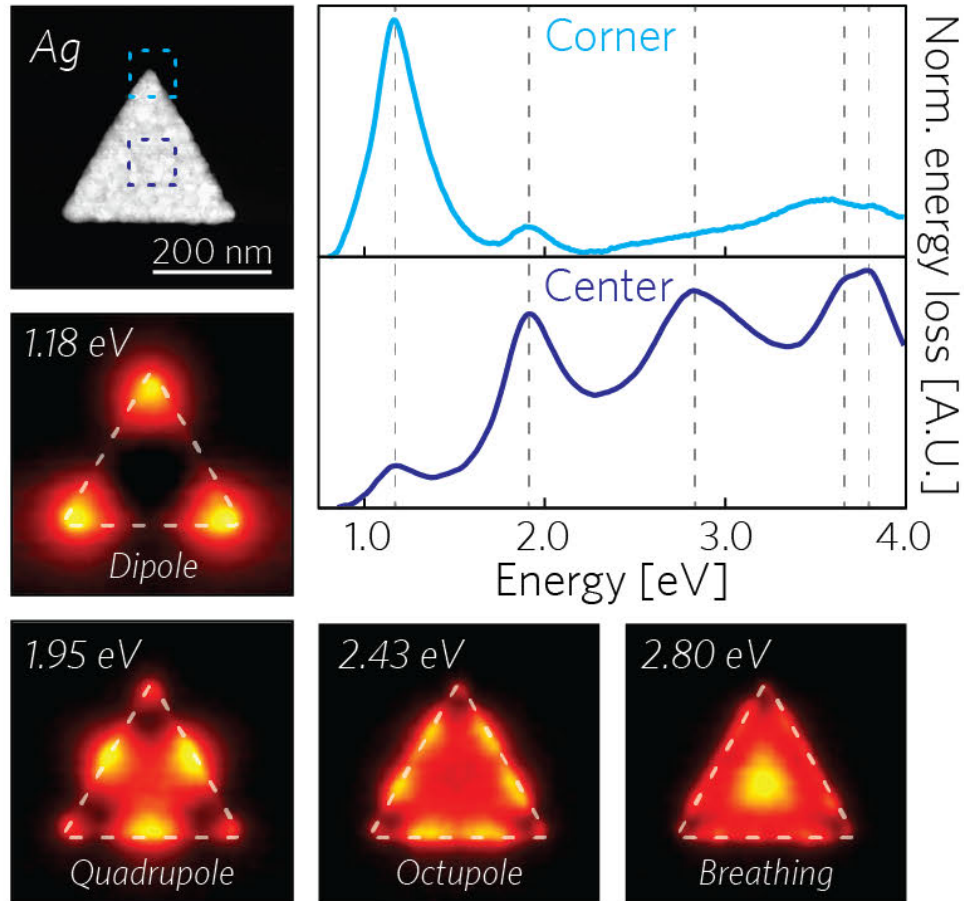
V. Flauraud et al. *ACS Nano* **11** (2017) 3485–3495

Rossow and Botton *PRL* **110** (2013) 066801

Mkhitarian *Nano Lett.* **21** (2021) 2444–2452

EPFL LSPRs on nanotriangle

- EELS hotspots for nodes in multipolar edge modes



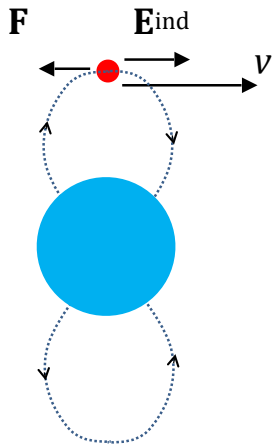
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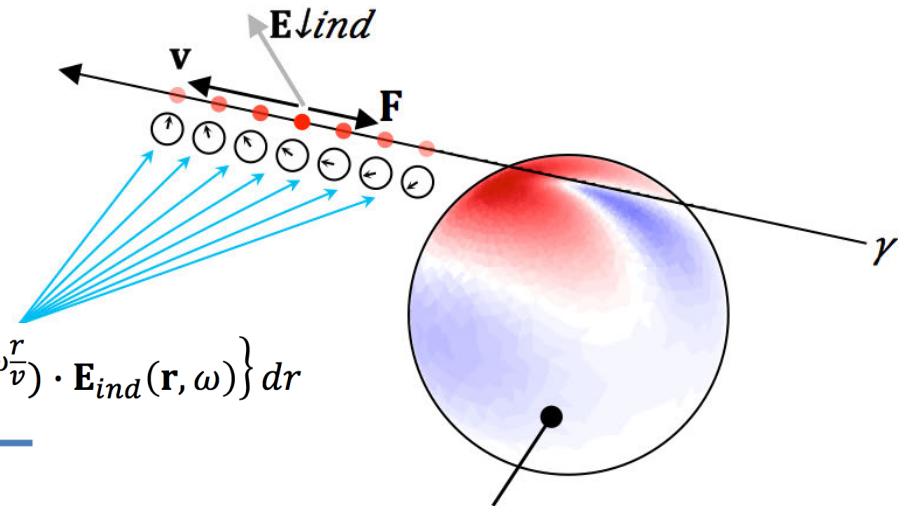
EPFL Classical: solve Maxwell's equations

- Take case of LSPR excitation on a metallic sphere
- Energy loss probability $\Gamma_{\text{EELS}}(\omega)$ related to work done by Lorentz force acting back on fast transmitted e^- from $\vec{E}_{\text{ind}}$

$$\Delta E = e \int dt \mathbf{v} \cdot \mathbf{E}^{\text{ind}}[\mathbf{r}_e(t), t] = \int_0^\infty \hbar \omega d\omega \Gamma_{\text{EELS}}(\omega)$$



$$\Gamma(\omega) = \frac{q}{\pi \hbar \omega v} \int_{\gamma} \text{Re} \left\{ \left(\mathbf{v} e^{-i\omega \frac{\mathbf{r}}{v}} \right) \cdot \mathbf{E}_{\text{ind}}(\mathbf{r}, \omega) \right\} d\mathbf{r}$$



Charge distribution at one frequency

EPFL Classical approaches/numerical methods

- Analytical solutions: exist for restricted geometries (e.g. spheres).
- Discrete-dipole approximation (DDA): treat nanoparticle as discrete assembly of point dipoles which interact with each other. Time consuming.
- Finite difference time domain (FDTD) method: solve full Maxwell equations in temporal domain, throughout space (3D). Less accurate than BEM.
- Boundary element method (BEM): use boundary conditions to reformulate Maxwell equations as a boundary integral equation (BIE). Fast computation but sensitive to complex parameterisation.
- Surface integral equation (SIE) method: Green's function approach to solve the EM wave equation. Restriction to isotropic/homogeneous media but versatile surface discretisation. Developed for EELS by NAM, EPFL.
- Photonic local density of states (LDOS): context for “fundamental” interpretation

EPFL Boundary element method

- Considering LSPRs: in quasi-static limit of $kL \ll 1$ where L is nanoparticle size, retardation can be neglected and wave eq. reduces to Poisson's eq.:

$$\nabla^2 \phi(\mathbf{r}, \omega) = -4\pi \rho(\mathbf{r}, \omega)$$

- Applying boundary relations this reduces to boundary integral eq.:

$$\Lambda(\omega)(\mathbf{s}) = \mathcal{P} \oint_S F(\mathbf{s}, \mathbf{s}') \sigma(\mathbf{s}')$$

where have constant: $\Lambda(\omega) = 2\pi \frac{\epsilon_d(\omega) + \epsilon_m(\omega)}{\epsilon_d(\omega) - \epsilon_m(\omega)}$

and kernel $F(\mathbf{s}, \mathbf{s}') = \frac{-\mathbf{n}(\mathbf{s}) \cdot (\mathbf{s} - \mathbf{s}')}{|\mathbf{s} - \mathbf{s}'|^3}$

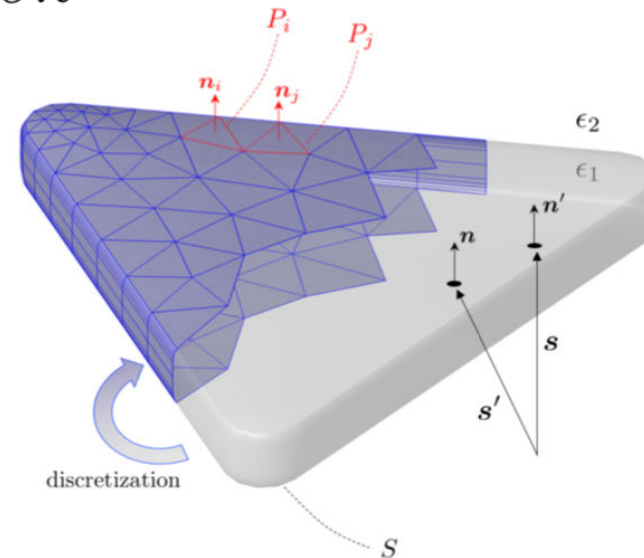
- Solutions that form infinite set $\{\lambda_n, \sigma_n\}_{n \in \mathbb{N}}$ and are solutions of eigenproblem

EPFL Boundary element method

- Boundary element method (BEM) extends this by applying EM perturbation described by scalar potential ϕ^{ext} giving:

$$\Lambda(\omega)\sigma(\mathbf{s},\omega) = \mathcal{P} \oint_S F(\mathbf{s},\mathbf{s}')\sigma(\mathbf{s}',\omega)d\mathbf{s}' + \frac{\partial\phi^{\text{ext}}(\mathbf{s}',\omega)}{\partial\mathbf{n}}$$

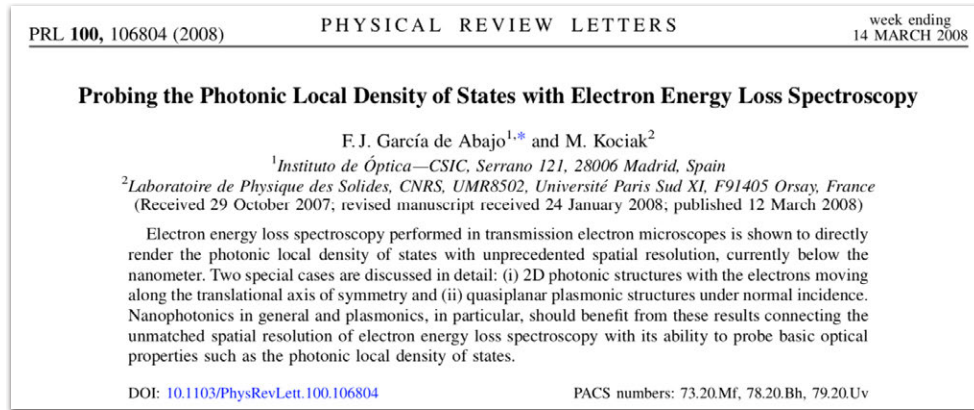
- Calculations by discretising surface and applying appropriate parameterisation



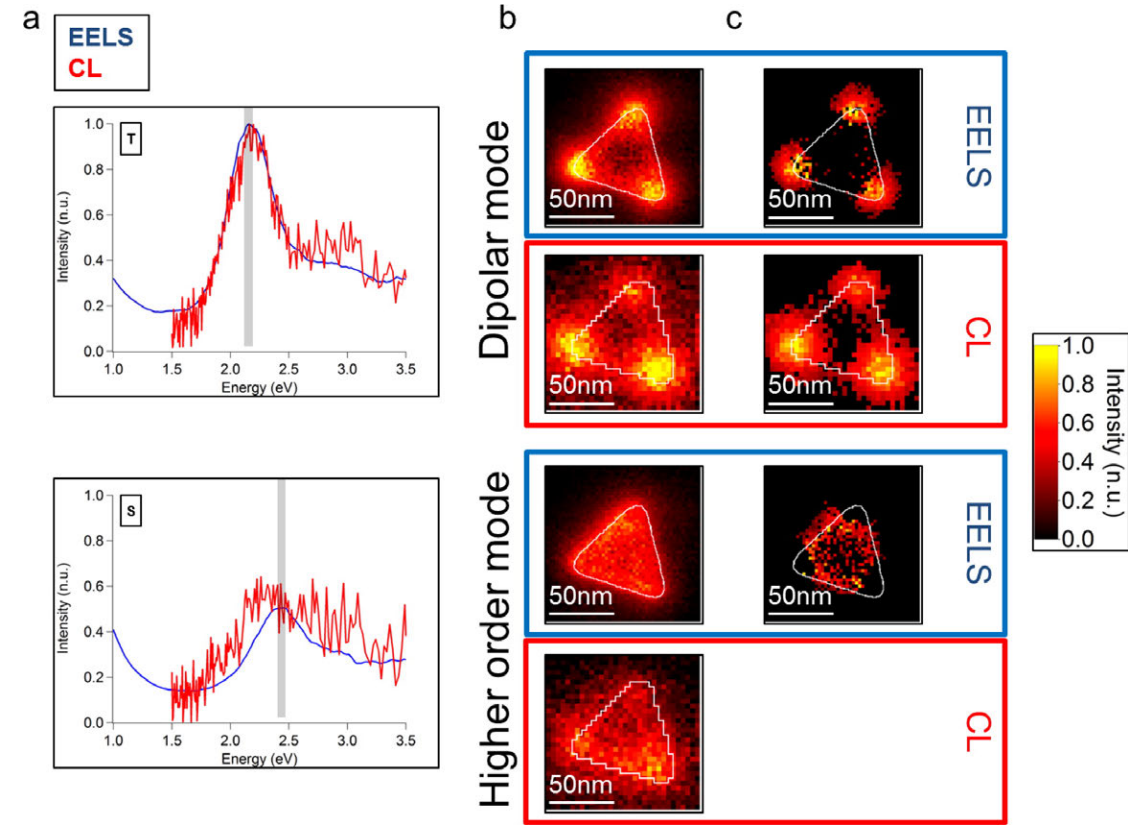
- Popular tool for applying BEM is MNPBEM toolbox by Hohenester and Trügler
<https://physik.uni-graz.at/de/mnpbem/>

EPFL Photonic local density of states

- García de Abajo and Kociak showed EELS related to full photonic local density of states (LDOS) of a nanostructure, including radiative & non-radiative components



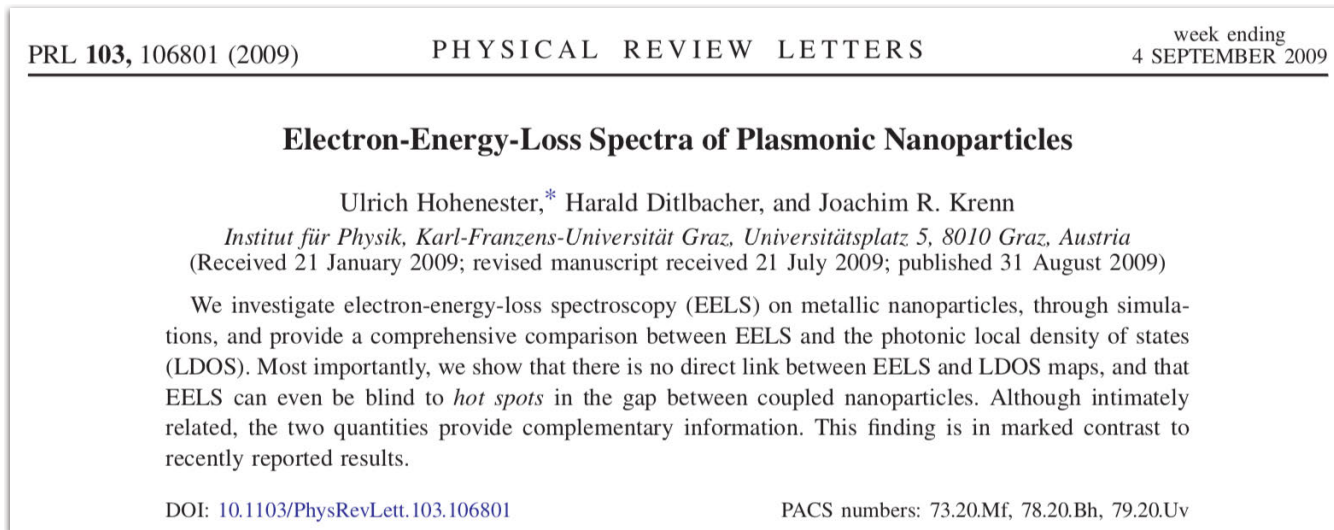
- Used in interpretation of correlated EELS and cathodoluminescence (CL) spectra; CL related only to radiative LDOS



STEM-CL and EELS maps of Ag nanotriangle:
A. Losquin et al. Nano Lett. **15** (2015) 1229

EPFL Photonic local density of states

- *However:* Hohenester et al. showed that, while they are intimately related, there is no direct link between EELS and LDOS maps.



- Importantly: EELS probes only component of LDOS parallel to e^- trajectory – scalar product of e^- velocity and $\vec{E}_{\text{ind}}$
- This can make it blind to EM hotspots between coupled nanoparticles

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EPFL Coupling of plasmonic nanoparticles

- Example: coupling of Au nanorods with different separations
- Unlike illumination using planewave EM radiation:
 - observe longitudinal anti-bonding mode; a *dark* mode
 - do not observe strong excitation of longitudinal bonding mode in the gap

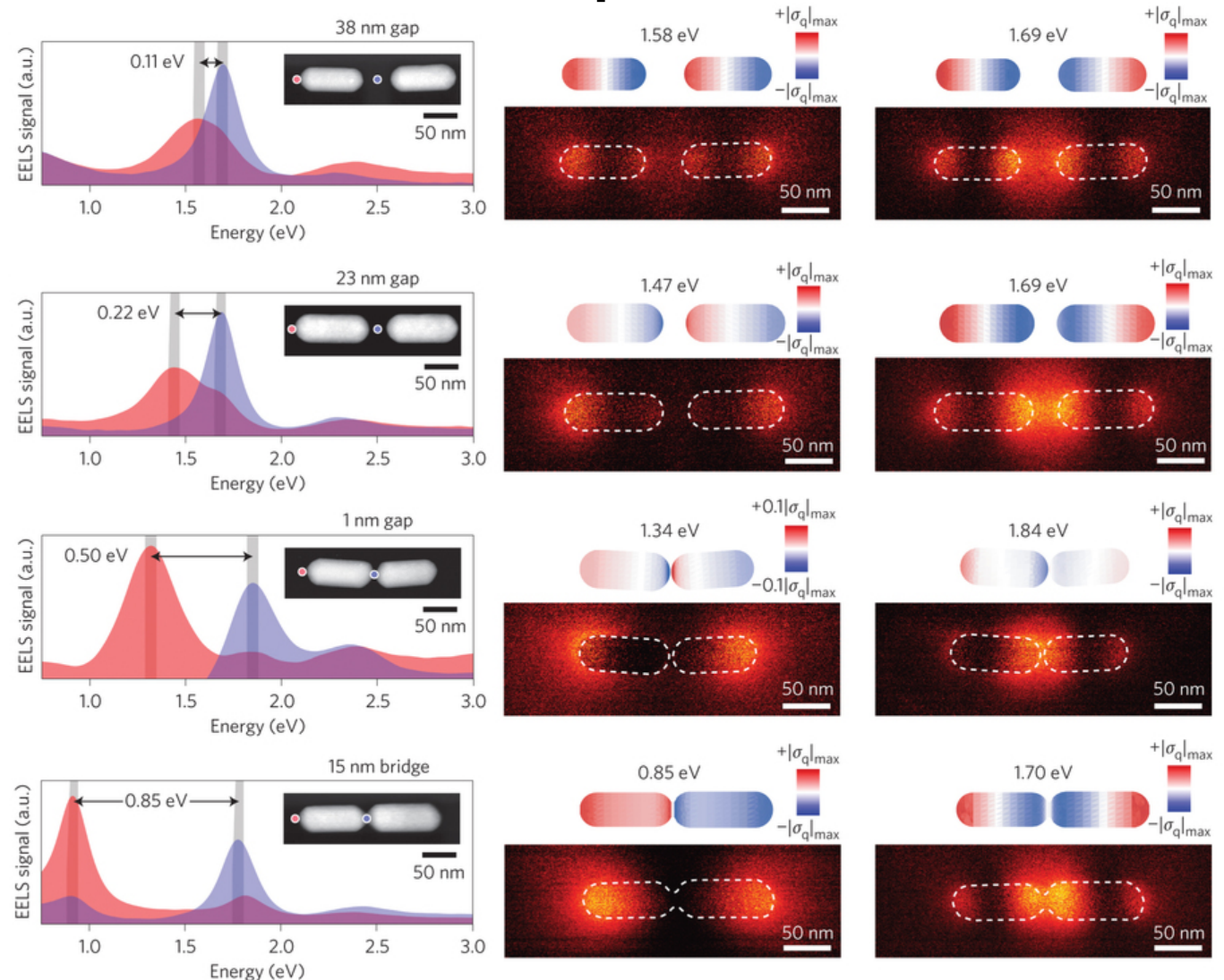
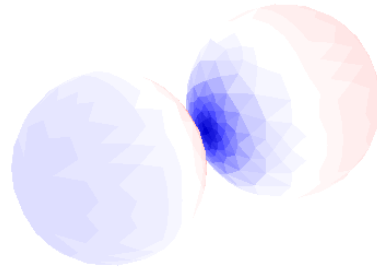


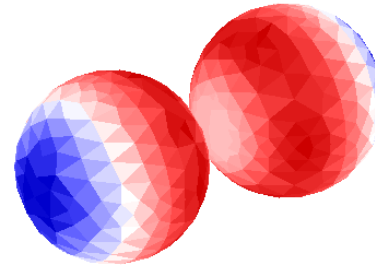
Figure from: V. Flauraud et al. Nature Nano **12** (2017) 73–80

EPFL EELS vs planewave excitation

- Planewave light: monochromatic and spatially extended
- Fast e^- : strongly localised, radially-symmetric electromagnetic field with broad spectrum $\Rightarrow$ near-field imaging over broad frequency range
- Consider possibility to excite different resonances/charge distributions from a dimer pair of small plasmonic spheres with different geometries of incident field:



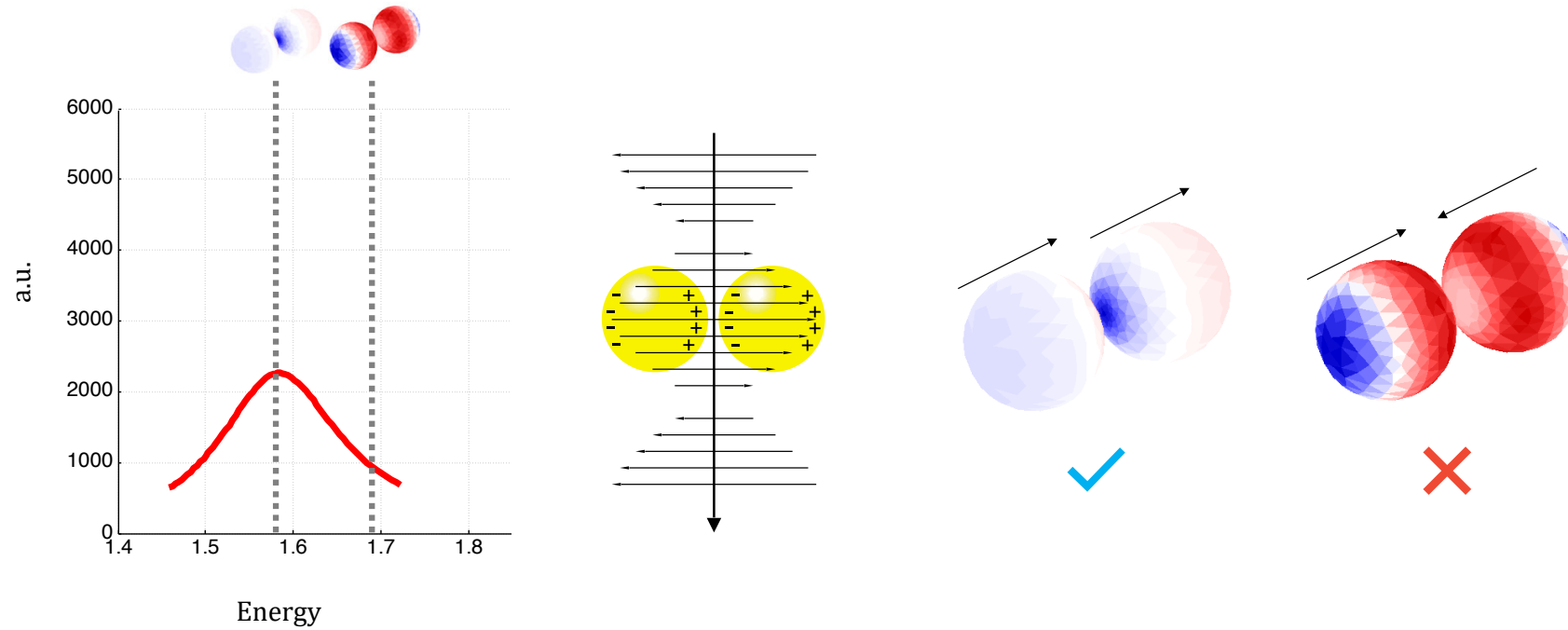
Longitudinal bonding



Longitudinal antibonding

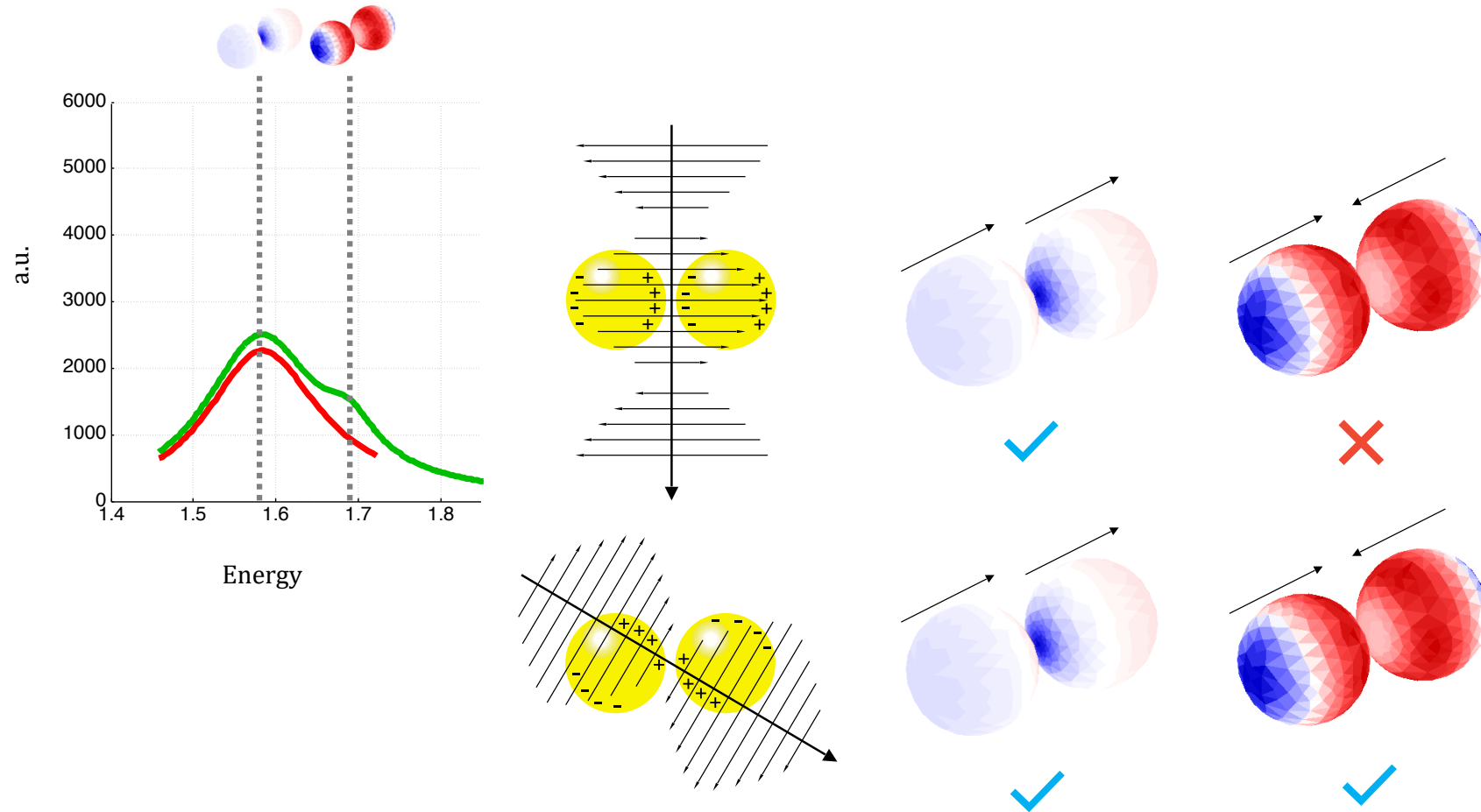
Figures courtesy of Gabriel D. Bernasconi
Formerly NAM, EPFL

EPFL Planewave excitation



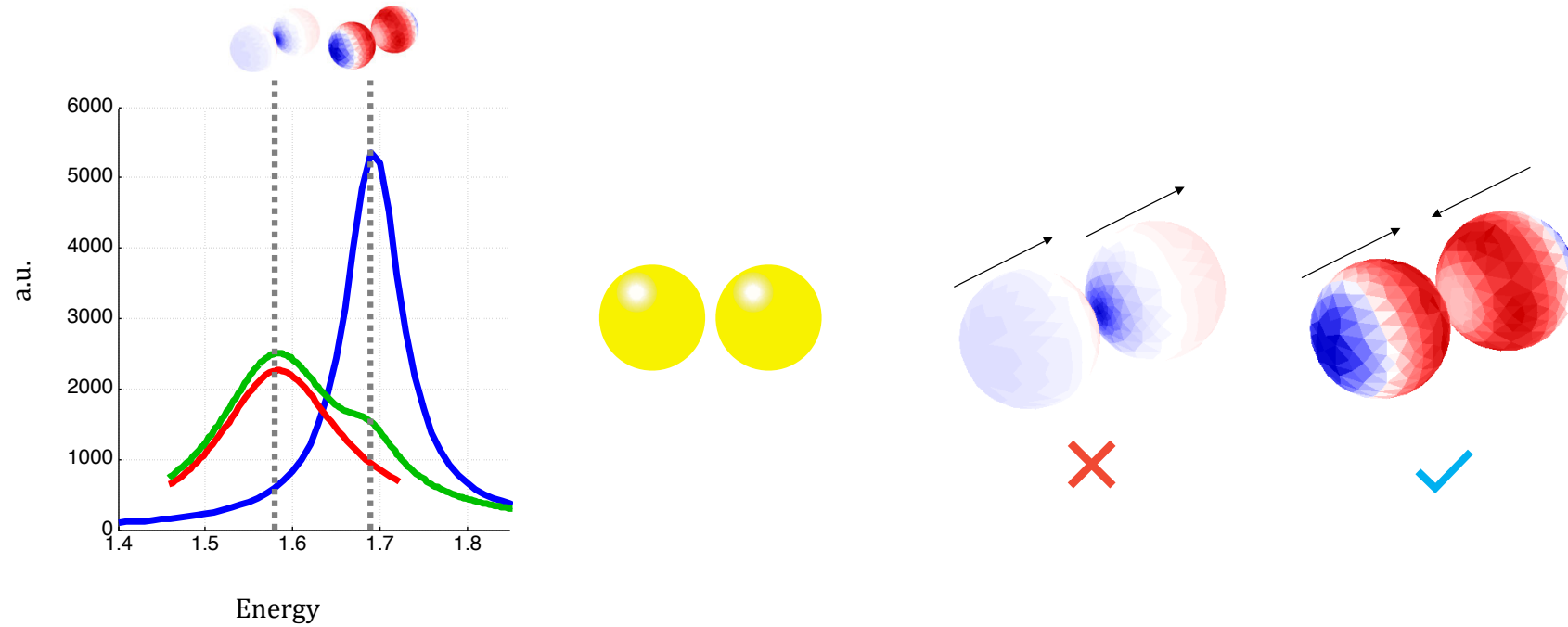
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EPFL Planewave excitation



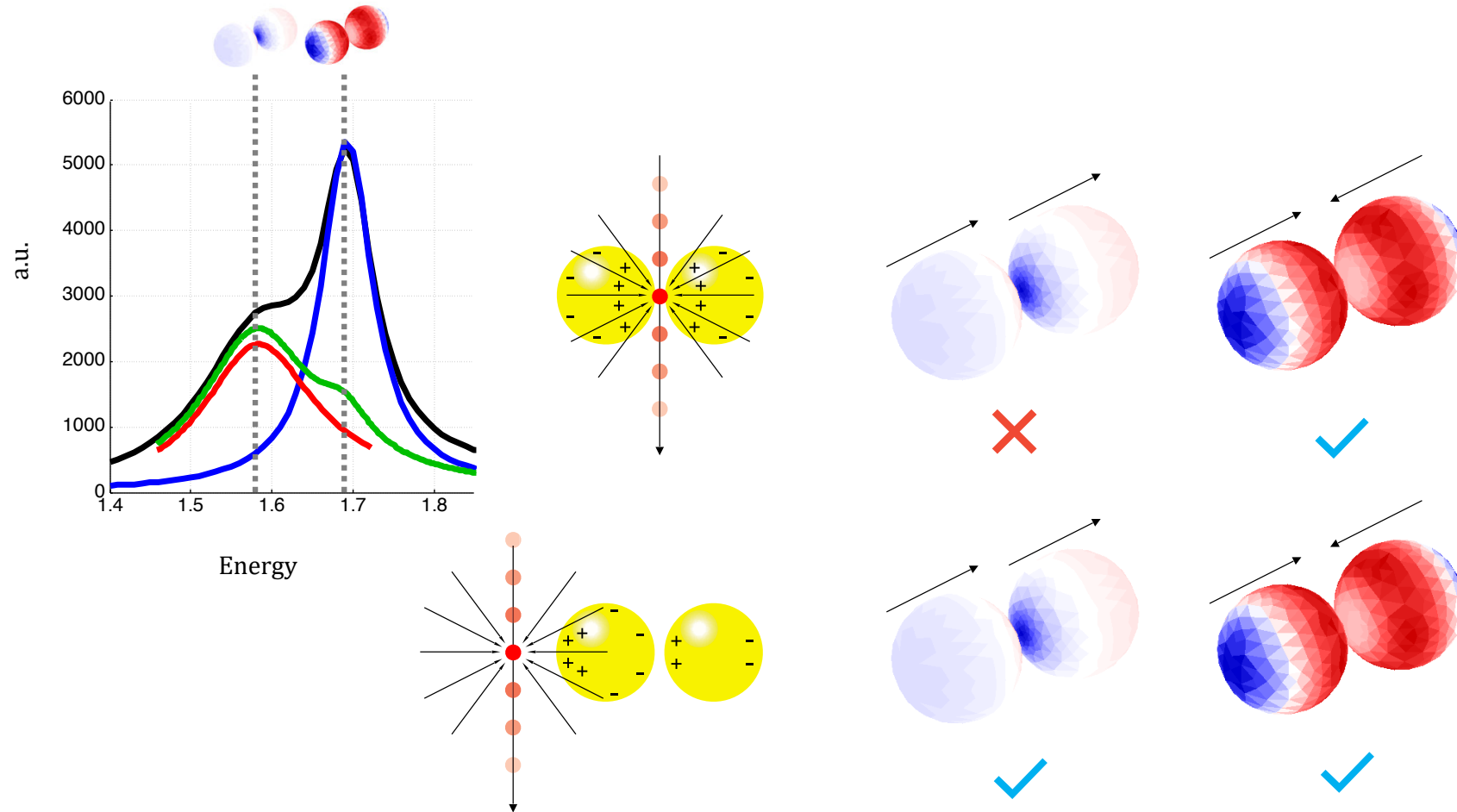
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EPFL Electron excitation



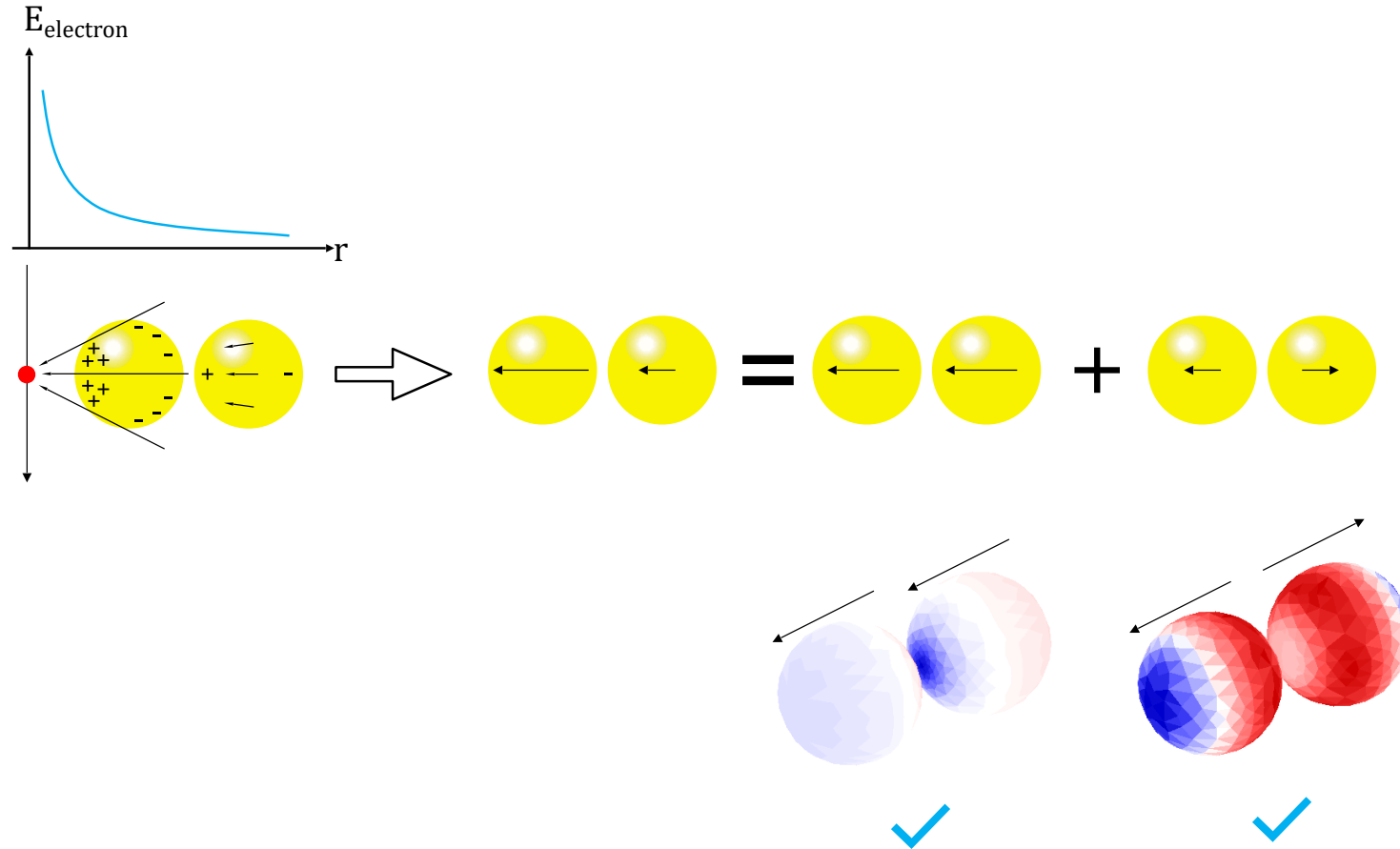
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EPFL Electron excitation



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Formerly NAM, EPFL

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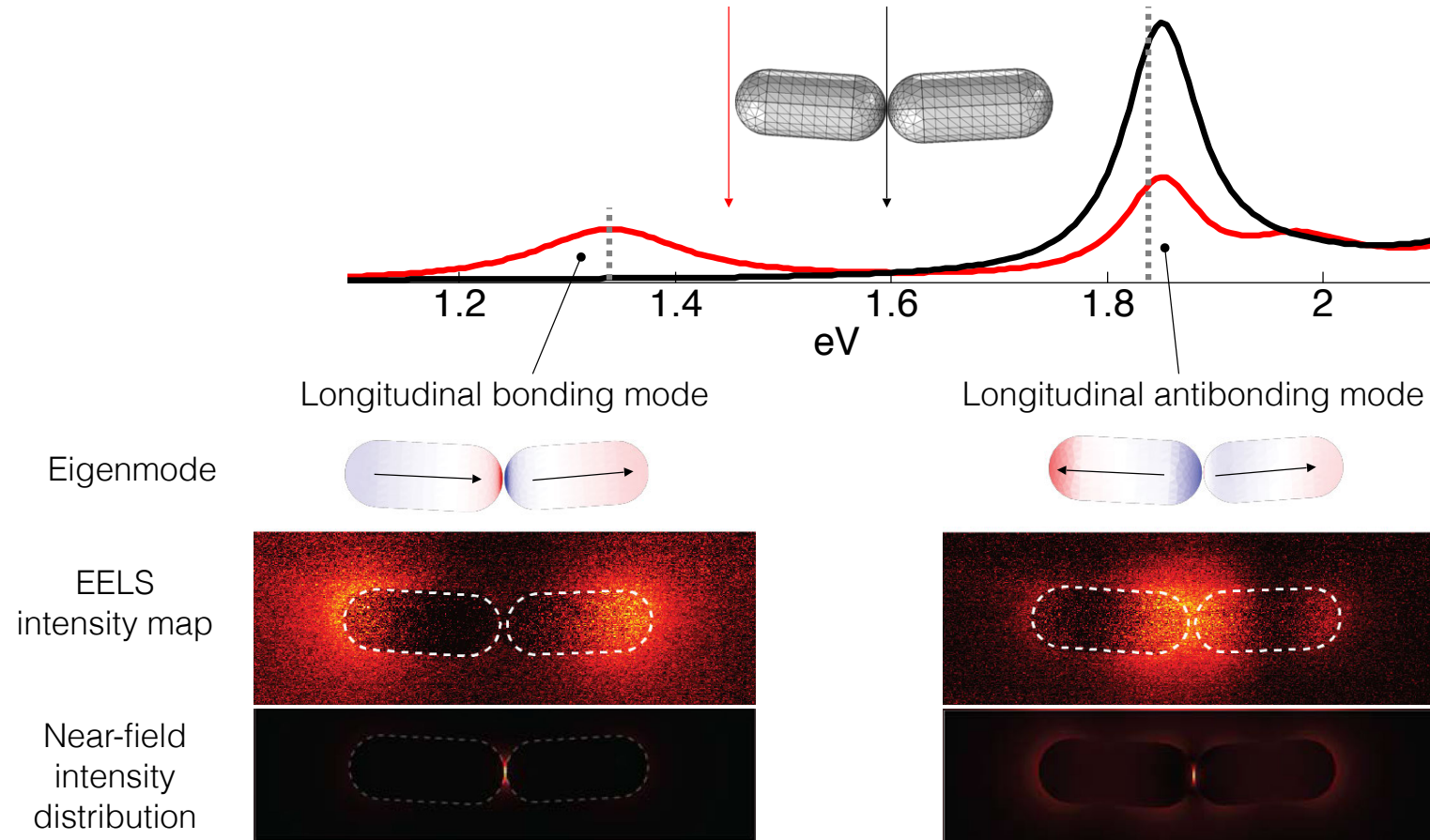


Figure by Gabriel D. Bernasconi et al. NAM, EPFL
EELS data: V. Flauraud et al. Nature Nano **12** (2017) 73–80

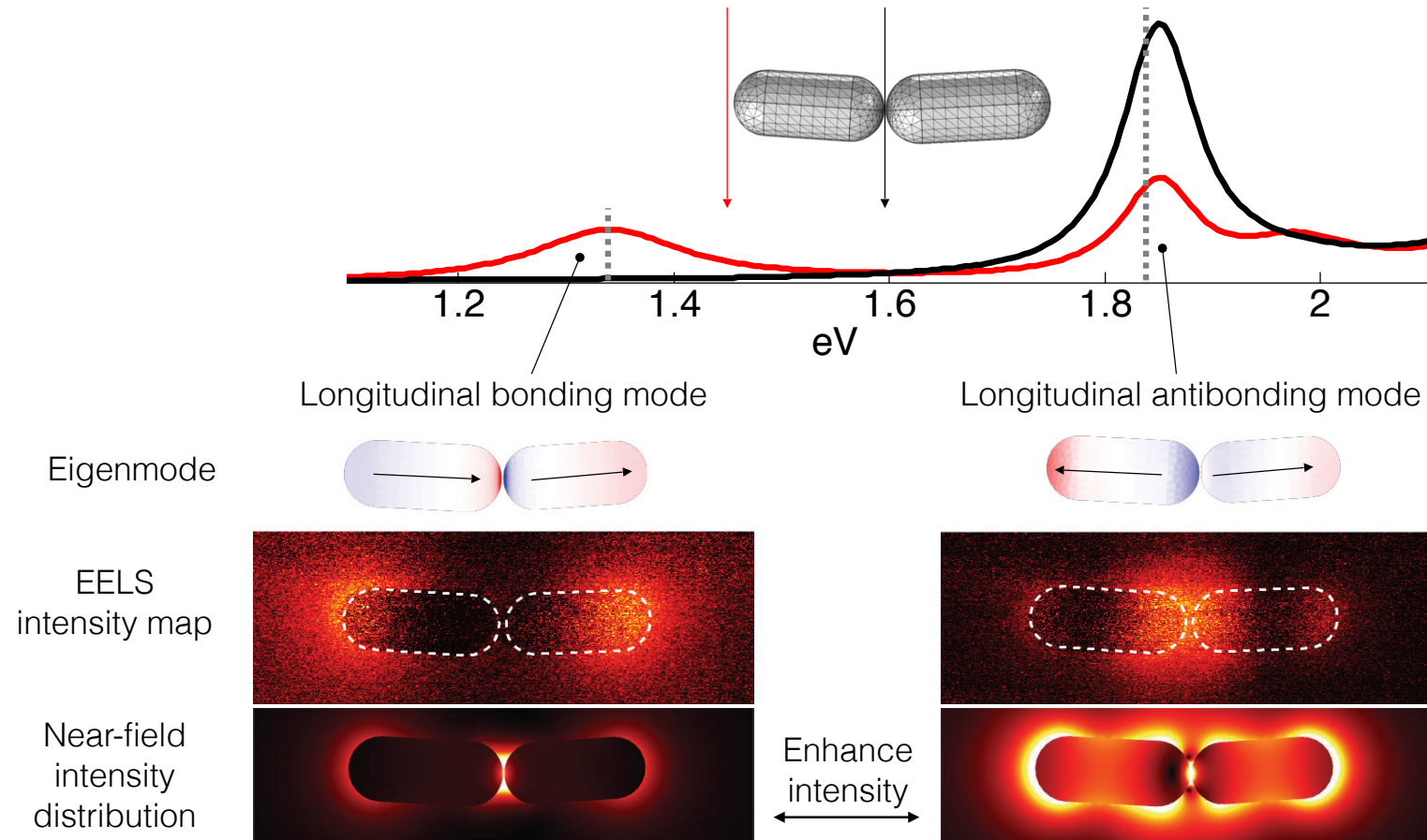


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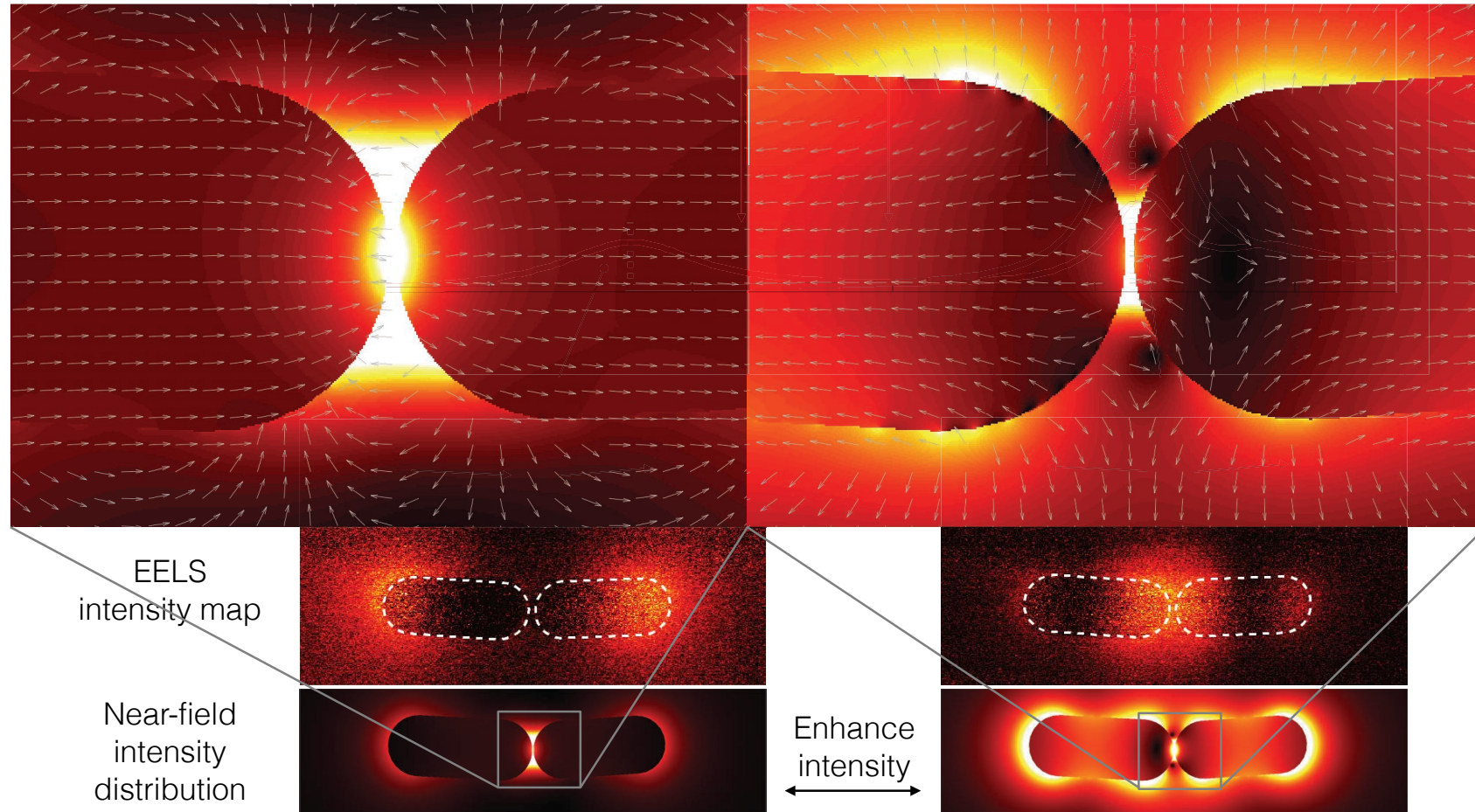
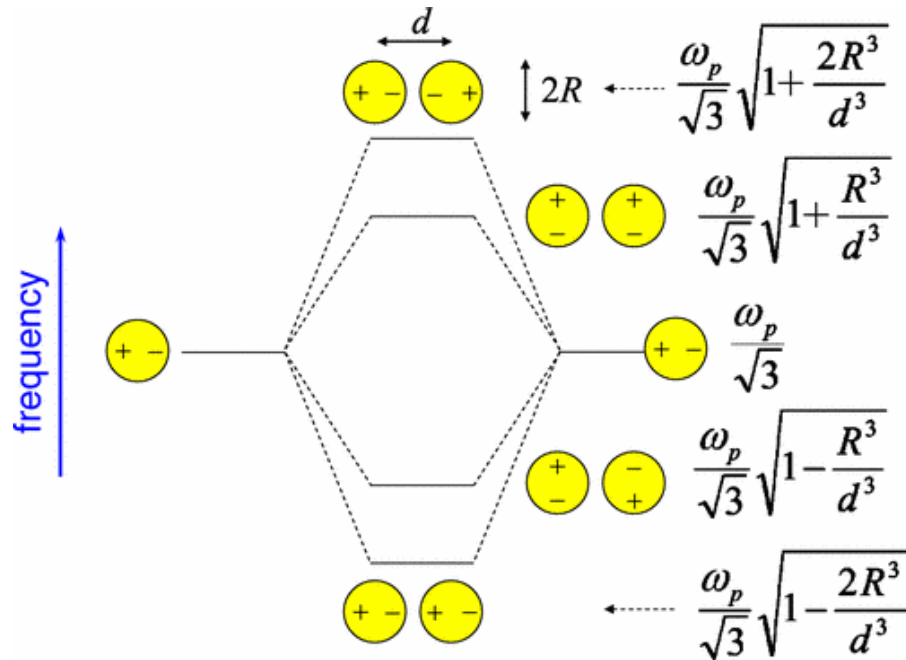


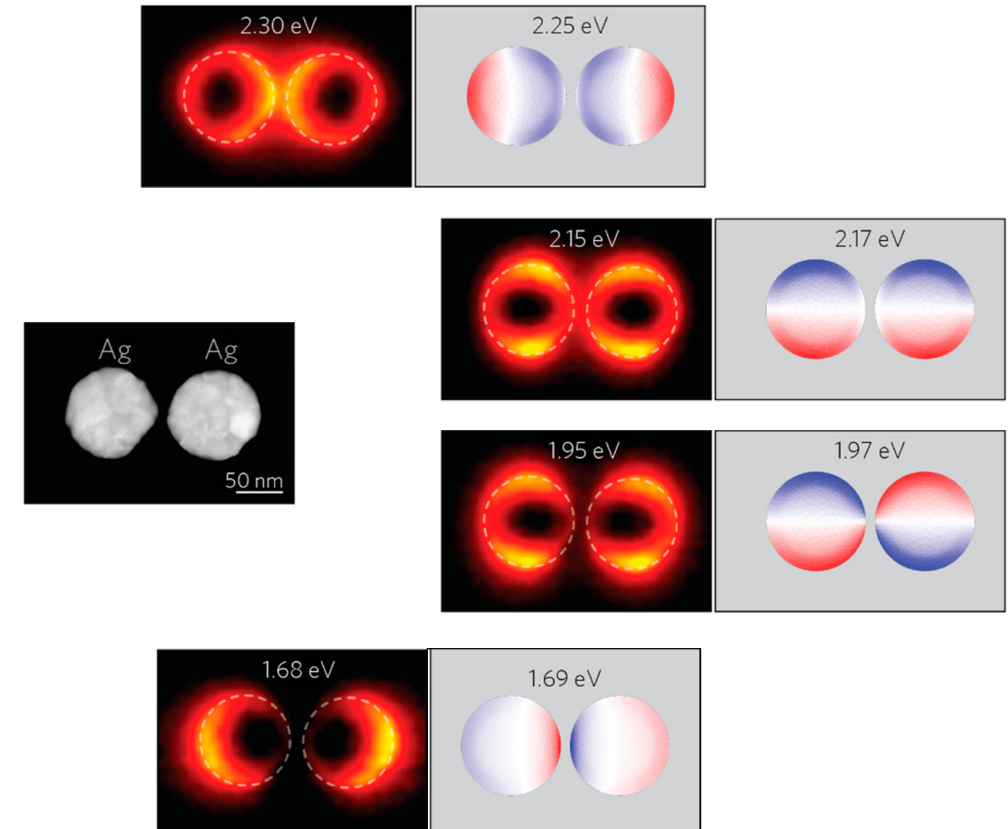
Figure by Gabriel D. Bernasconi et al. NAM, EPFL
EELS data: V. Flauraud et al. Nature Nano **12** (2017) 73–80

EPFL Dimer hybridisation

- From one dipole mode, dimer hybridisation of a radially symmetric nps $\Rightarrow$ 4 modes
- EELS permits measurement of all of them
- Model (left): interacting spheres
- EELS data (right): Ag discs



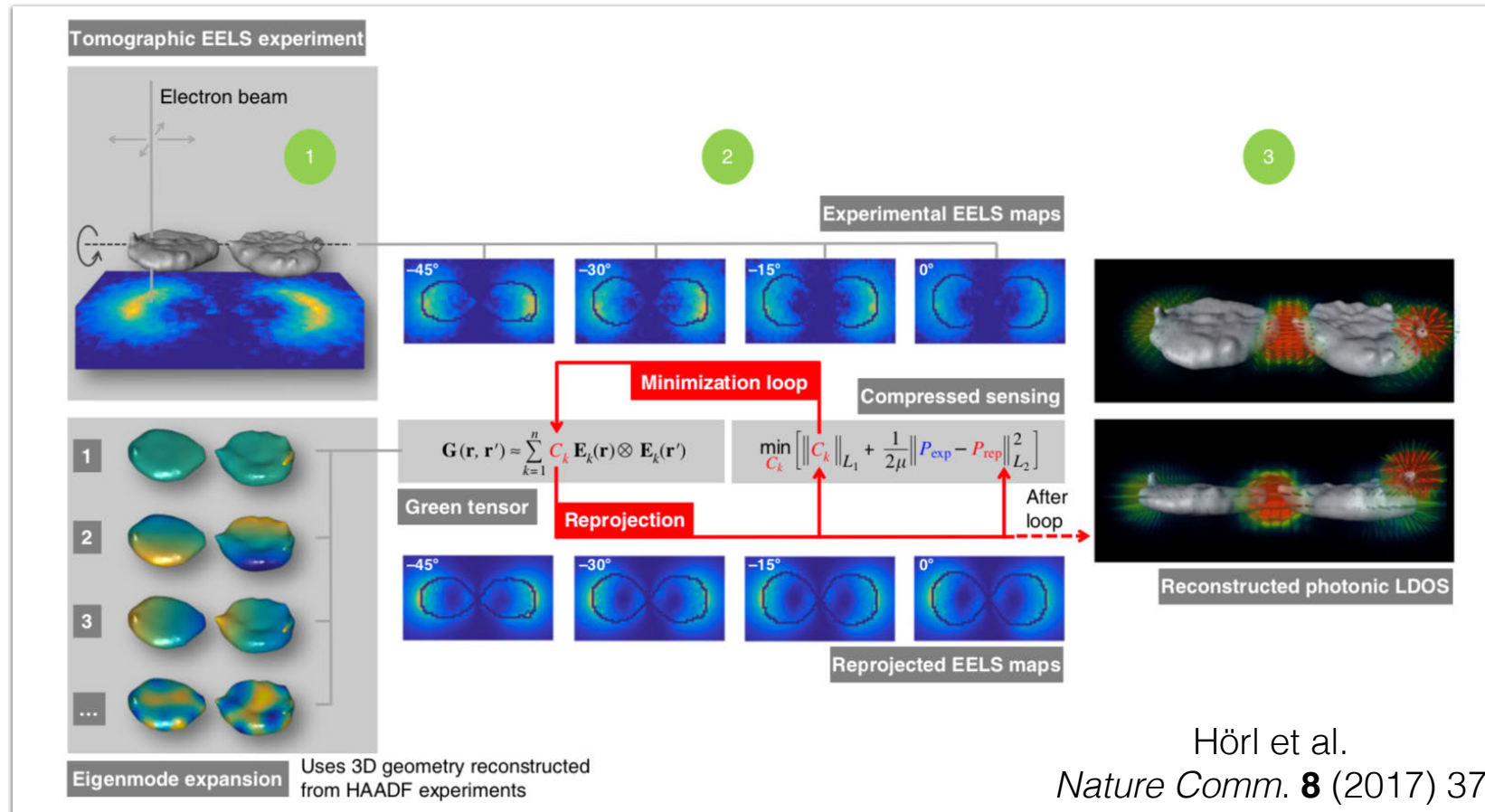
S. Riikonen et al. *PRB* **71** (2005) 235104



V. Flauraud et al. *ACS Nano* **11** (2017) 3485–3495

EPFL 3D imaging of bonding mode

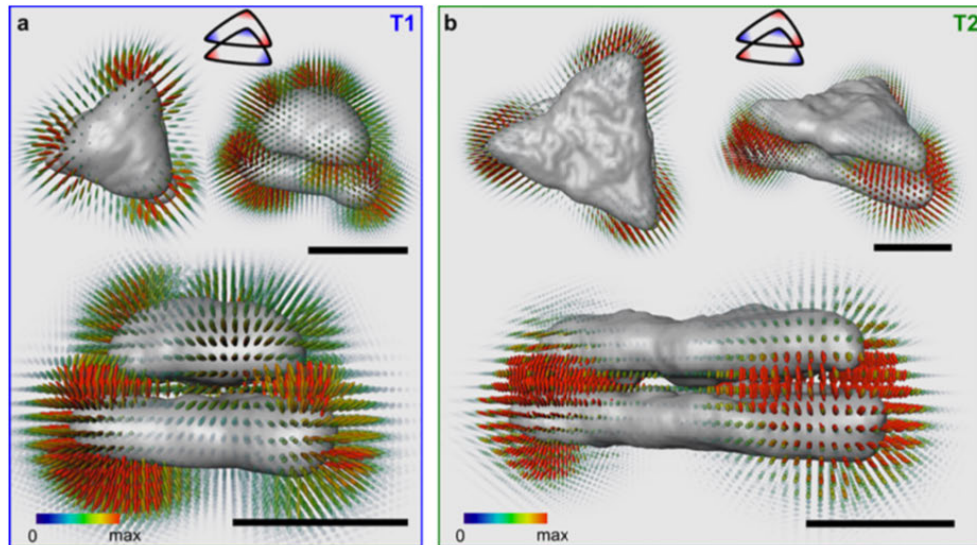
- Hörl et al. developed tomographic approach to imaging photonic environment of plasmonic nanoparticles through solution of an inverse problem



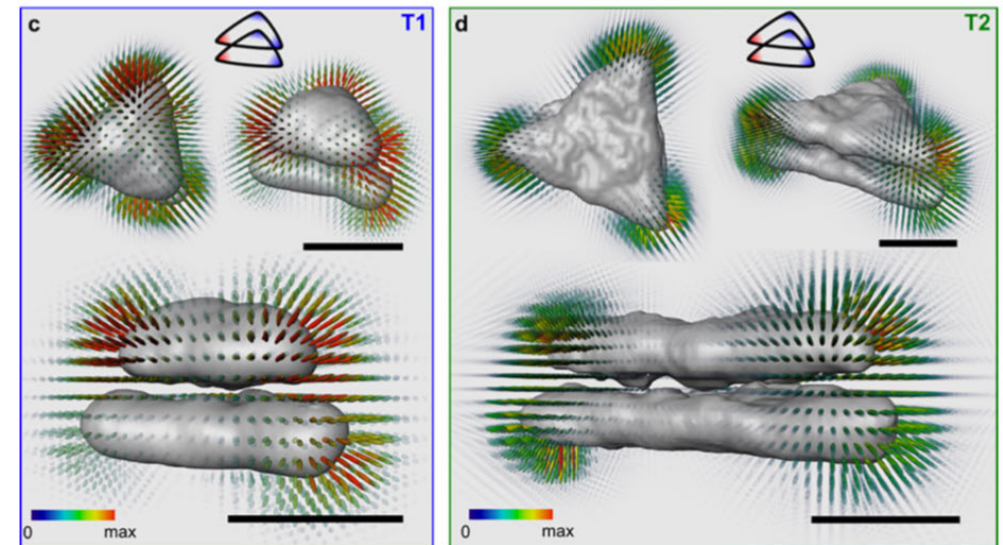
EPFL 3D imaging of bonding mode

- Applied to vertically-stacked dimer pair has allowed the gap bonding mode to be directly visualised:

Bonding modes



Anti-bonding modes



Haberfehlner et al. *Nano Lett.* **17** (2017) 6773–6777

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EPFL Plasmon damping

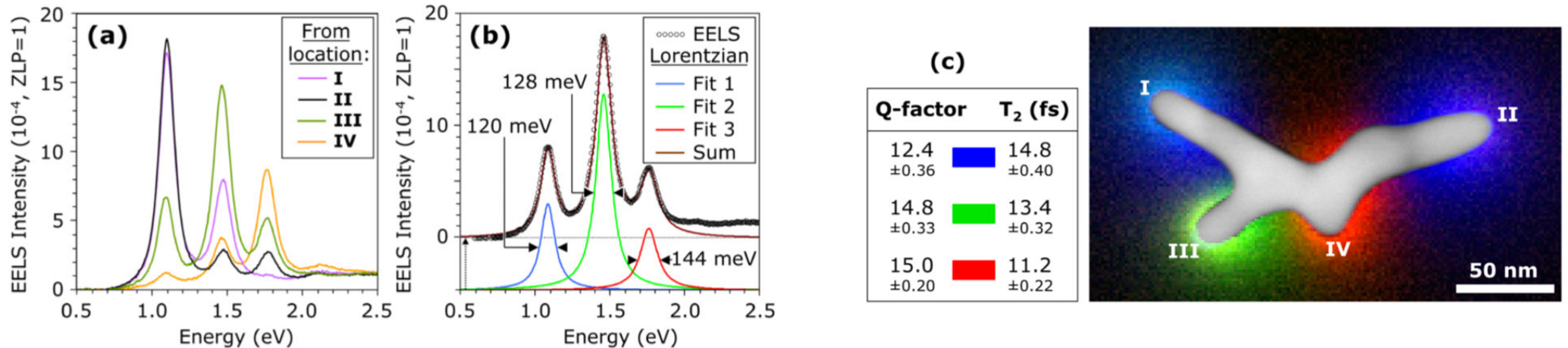
- LSPRs lose energy via:
 - scattering radiation to far-field (as could be measured by CL)
 - non-radiative channels, i.e. absorption, leading to thermal dissipation
- Decay time on the order of 10s of femtoseconds
- Can describe with damped oscillator model



EPFL Plasmon damping

- By carefully measuring width of plasmon peaks in EELS, Bosman et al. were able to measure the Q-factor and dephasing times T_2 for damping of different resonances in Au nanostructures:

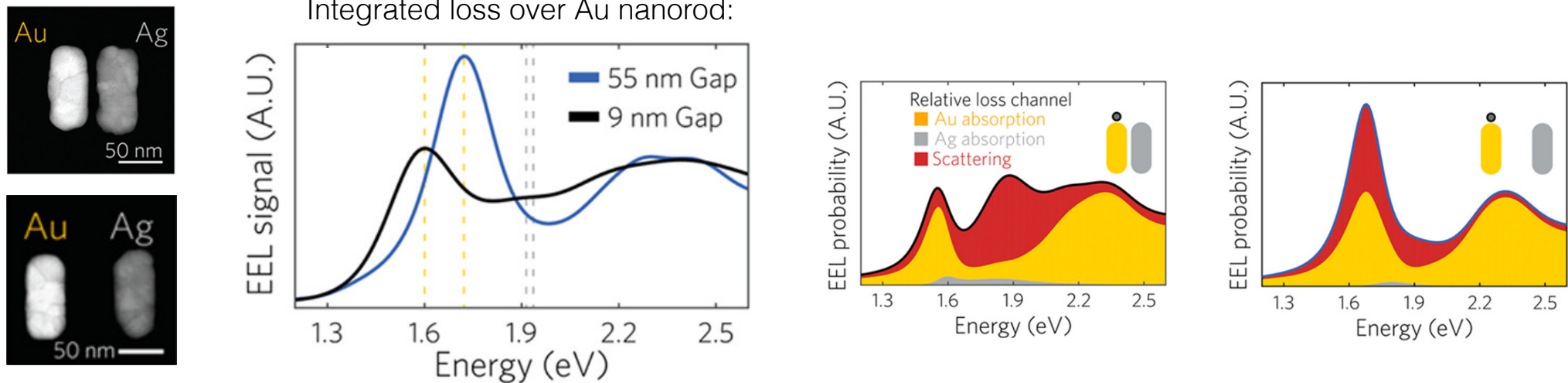
$$Q = \frac{E_p}{\Gamma}$$



Bosman et al. *Sci. Rep.* **3** (2013) 1312

EPFL Antenna theory for scattering & absorption

- Bernasconi et al. introduced alternative antenna-based approach to EELS
- By coupling energy conservation law with full-wave EM computations based on a SIE method, derive EEL spectra directly from dissipative processes of scattering & absorption
- Reveals interplay between absorption and scattering; i.e. where energy goes



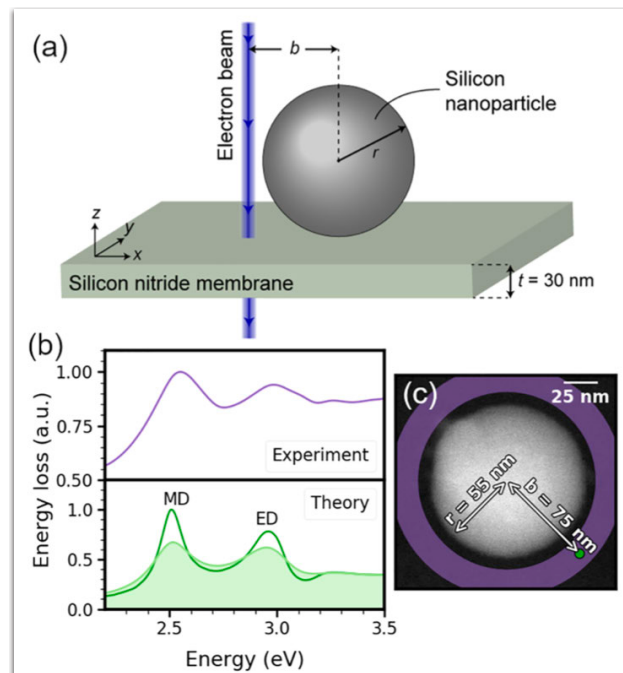
Bernasconi et al. *ACS Photonics* **4** (2017) 156–164; V. Flauraud et al. *ACS Nano* **11** (2017) 3485–3495

EPFL Contents

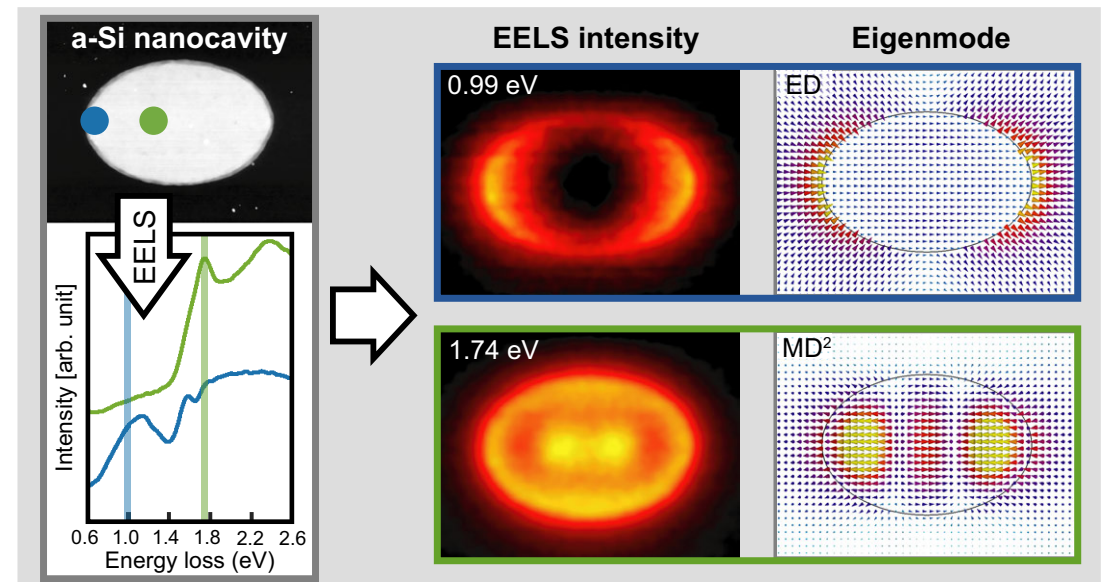
- Kramers-Kronig analysis and band gap measurements in visible light regime
- An evanescent probe
- Introduction to localised surface plasmon resonances (LSPRs) in EELS
- Theoretical approaches and numerical methods for EELS LSPR analysis
Dielectric in optical regime
- Coupling of plasmonic nanoparticles: EELS vs planewave illumination;
imaging of hybridisation and bonding modes
- Plasmon damping; scattering and absorption
- Dielectrics and bound-states-in-the-continuum

EPFL Dielectric nanostructures

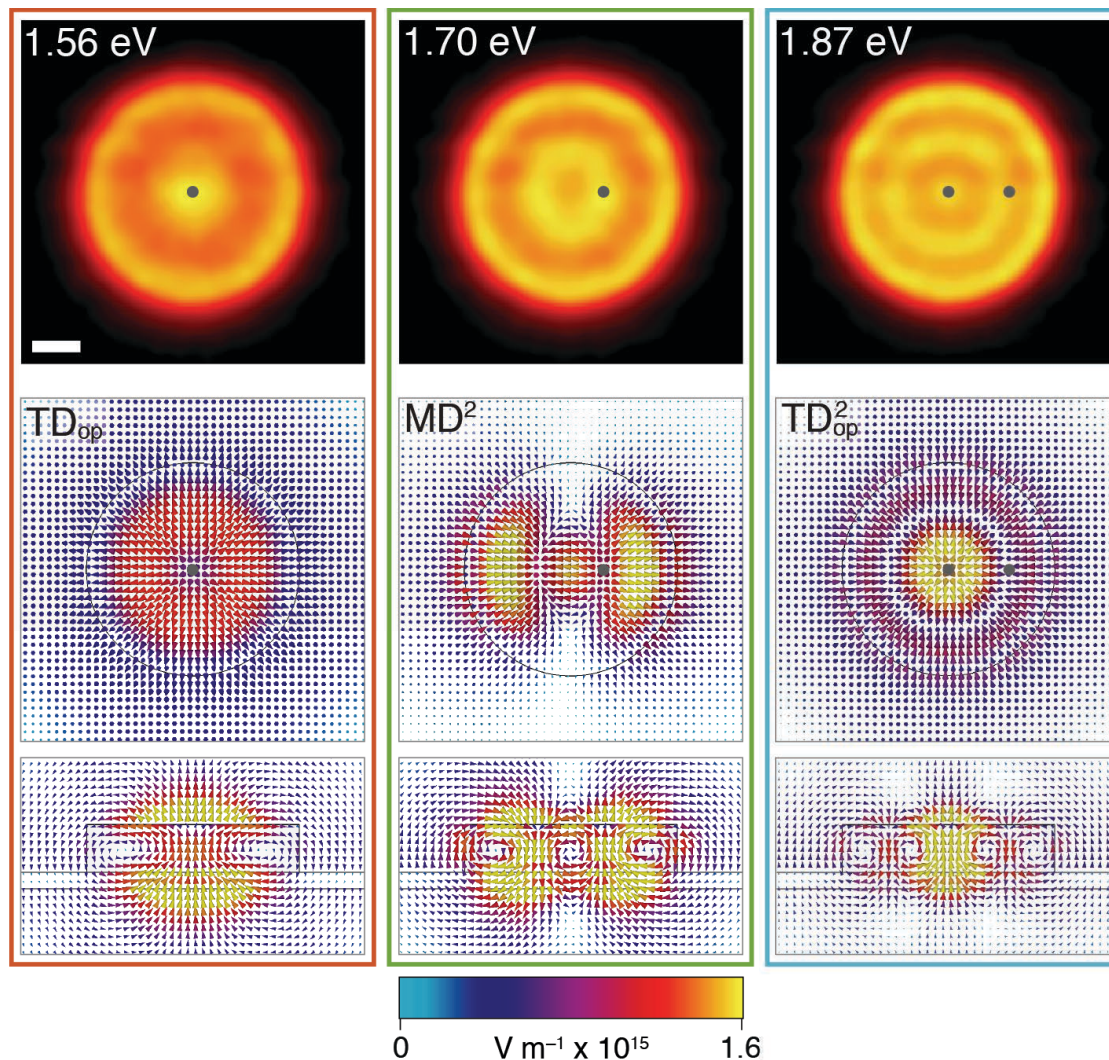
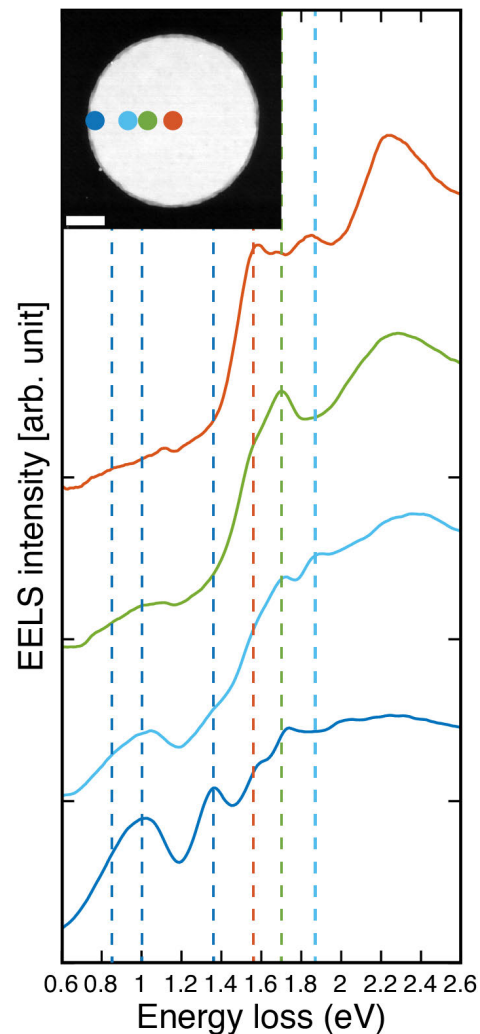
- Dielectric nanostructures that support Mie-type resonances also of interest for nanophotonics applications
- Resonances correspond to charge displacement currents in particle volume
- Resonances can be excited by e^- probe and measured by EELS analogously to LSPRs



Assadillayev et al. *ACS Photonics* **8** (2021) 1582

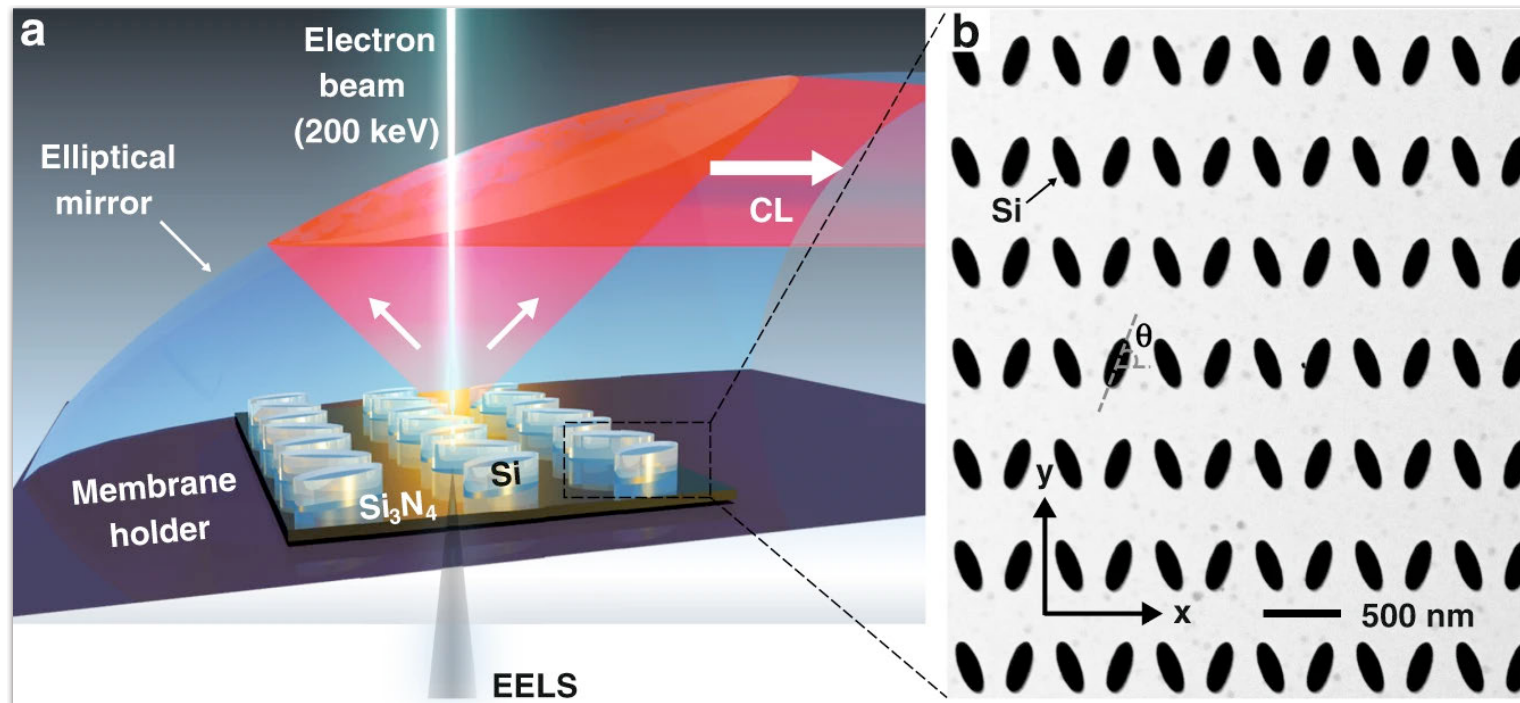


Alexander et al. *ACS Nano* **15** (2021) 16501



EPFL Bound-states-in-the-continuum (BIC)

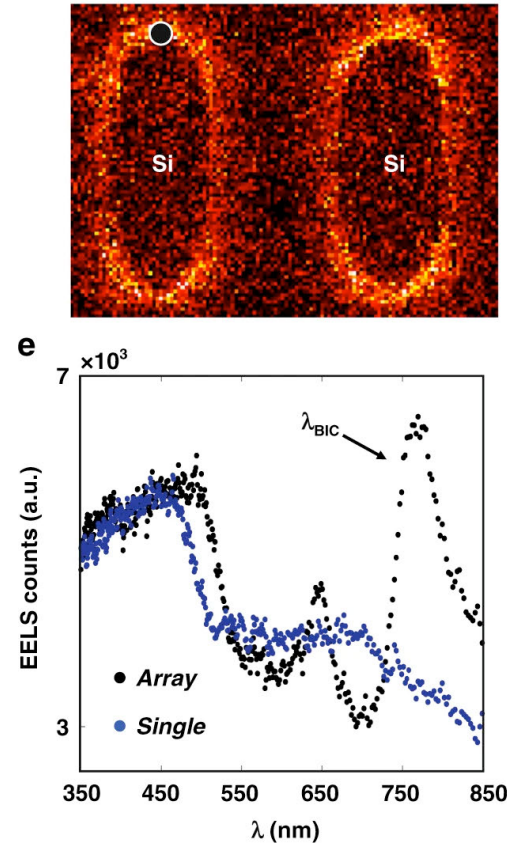
- Optical modes with energies higher than potential wells that are still localised and bound in space $\Rightarrow$ complete suppression of radiative loss in far-field
- Dong et al. studied BIC with EELS (near-field) and CL (far-field)



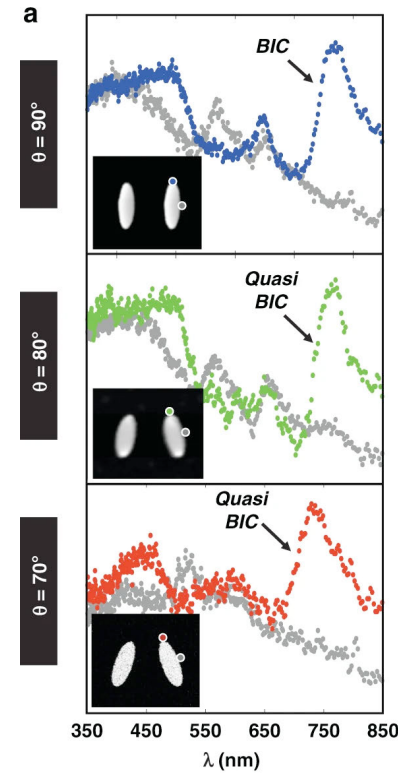
Si elliptic cylinder nanoantennas that support (quasi-) BIC modes

EPFL Bound-states-in-the-continuum (BIC)

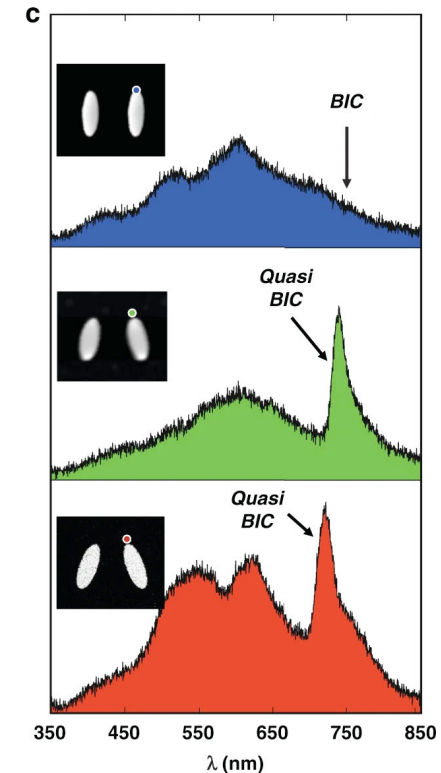
- BIC excitation only seen for *array* of nanoantennas
- Tilted nanoantennas: BIC loses full confinement, some radiation escapes to far-field (Quasi BIC)



EELS



CL

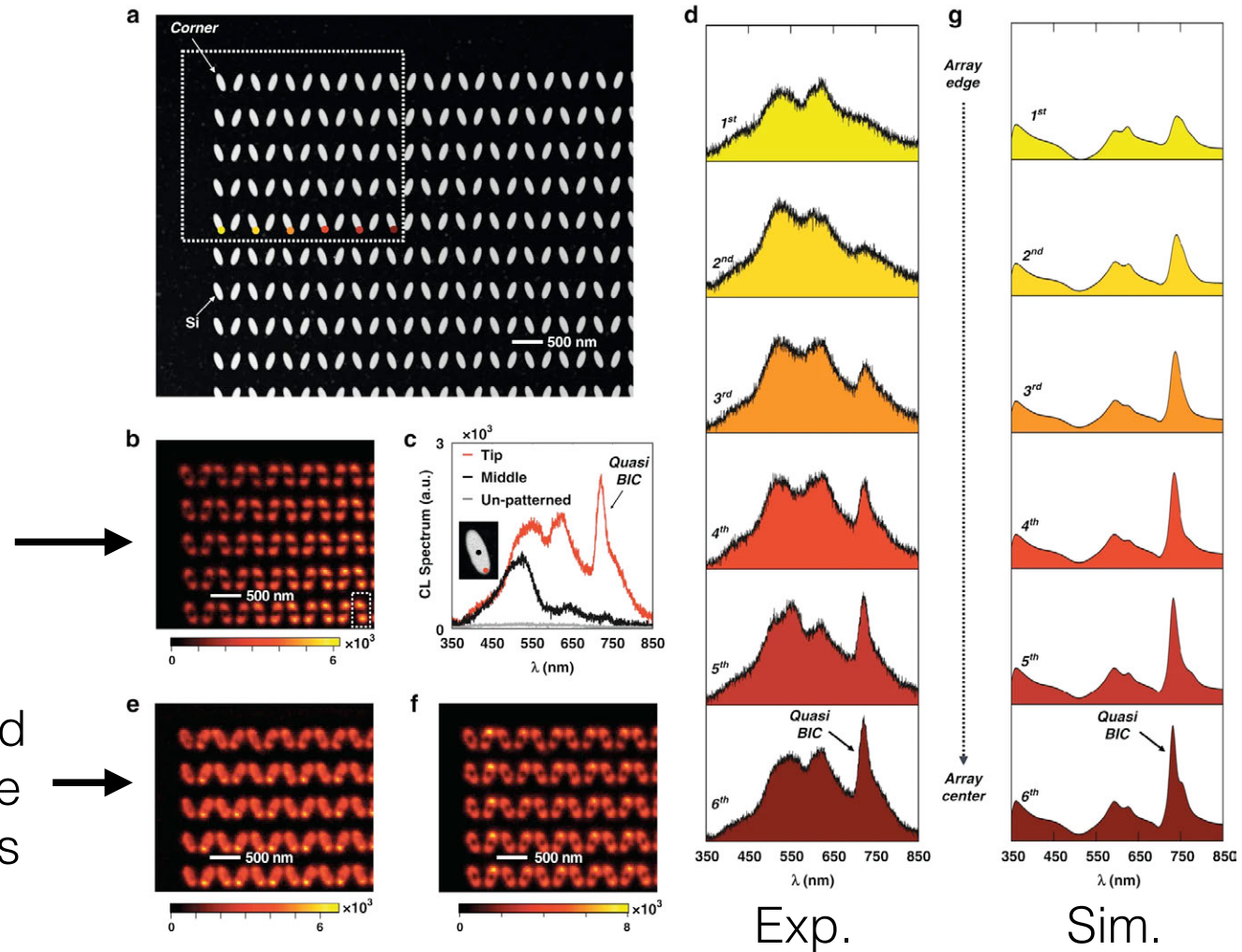


EPFL Bound-states-in-the-continuum (BIC)

- Cathodoluminescence measurements across Quasi BIC array

Quasi BIC frequency

522 nm and 614 nm Mie resonances



EPFL Not discussed today...

- Relativistic quantum electrodynamic approach: Hugo Lourenço-Martins et al. *SciPost Phys.* **10** (2021) 031. E.g. for holography EELS and other phase-related experiments
- Probing LSPRs using phase-shaped electron beams. By matching wave function of e⁻-beam to plasmonic symmetry can selectively detect specific plasmon modes. HLM paper; Guzzinati et al. *Nat. Comm.* **8** (2017) 14999
- Study of excitations in dielectric photonic crystals. Conceptually: virtual broad band light from Cherenkov & transition radiation couples to photonic LDOS. García de Abajo et al. *PRL* **91** (2003) 143902, Cha et al. *PRB* **81** (2010) 113102 – TM modes measured in aloof geometry; Le Thomas et al. *PRB* **87** (2013) 155314 – cavity TE mode measured in bulk
- Incoherent radiation studied by EELS and STEM-CL – e.g. excitonic effects in EELS, radiation from e⁻-h⁺ exciton recombination at quantum dots; quantum emitters; ...